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CALCULATING EFFICIENT TEAM SIZE: BALANCING DECIDING AND DOING AS AN ELEMENTARY OPTIMIZATION PROBLEM

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ABSTRACT

This paper builds elementary models of efficient team size by balancing costs of *deciding* and *doing* that often exists in team settings. A simple model assuming linear decision time and inversely linear execution time is first constructed and optimized. That model is augmented by a concern for solution quality using simple probability calculations. Thereafter, the basic model is generalized somewhat by permitting decision making and doing to be governed by power-law terms. Then, the idea of using simple, one-dimensional optimization as a modeling tool in organizational contexts and elsewhere is formalized as an elementary optimization problem (EOP), and the characteristics of EOPs and some exemplar problems are enumerated. The paper concludes by suggesting how the systematic study of EOPs, other simple models, and patchquilt integration using dimensional analysis may permit the formulation of a more sophisticated quantitative understanding of problems in organizational theory than would otherwise be possible.

(**Keywords:** Teamwork, team size, optimization, patchquilt, integration.)

1 Introduction

Since the rise of modern *quality management* methods [1], firms have focused on the importance of *teamwork* as a way to improve decision making, coordinate complex activities, and speed execution. For example, Chrysler has dramatically improved its performance since the early 90s primarily due to the use of platform teams [2]. Later, Ford with their Ford 2000 Program [3], and GM with their multiple organizational changes including the use of vehicle teams are also viewed as taking similar paths aimed at harnessing the power of teams [4].

As modern corporations have focused on the role and importance of teams, the business and management aisles of bookstores have filled with volumes extolling the virtues of teamwork, but much of the research is empirical in orientation and qualitative in methodology. Engineering by its nature seeks quantitative understanding of its subjects, even those involving human beings, and here we build quantitative models of team size by trying to balance critical factors to a team's success.

In particular, we build models of efficient team size by optimizing the balance between *deciding* and *doing*. As many have observed, decision making becomes more costly as the number of decision makers increases [5], yet dividing a complex task among members of the team may permit that task to be completed sooner than otherwise might be possible.

The paper starts by reviewing key background information on teamwork and its modeling. It continues by developing the basic model of deciding and doing. The model is augmented to permit some understanding of the tradeoff between solution speed and quality. Thereafter the basic model is extended using power-law models and these permit quantitative understanding of situations where decision making is more or less complex and where increased team size results in either increased shirking or increased synergy between teammates. Finally, the very notion of using elementary optimization to construct quantitative models in organizational theory and elsewhere is formalized as an *elementary optimization problem* or EOP. EOPs are those that optimize a single variable, have a single optimum, and are analytically tractable. Although these limitations sound much too restrictive, the paper ends by suggesting how EOPs can be used with other simple or facetwise models and patchquilt integration using dimensional reasoning to build up fairly sophisticated understanding of complex organizational situations and other complex landscape with a minimum of algebra and a maximum of insight and understanding.

2 Background

The literature of teamwork is vast and growing, and here we merely touch on some of the types of literature available. Much of the influential literature today is social scientific in orientation combining relevant theories from organizational behavior as well as empirical observation to offer qualitative models of teams and teamwork and practical advice on team formation and building [6].

The early days of organizational theory were marked by more quantitative concern for the combinatorics of organizational relationships [7] and span of control [8], but a famous article by Herbert Simon [9] called into question many of the principles of scientific management [10], and simplified analytical approaches to understanding organizations fell out of favor [11].

More recently, more complex mathematical-computational approaches have arisen, and these seek organizational understanding through fairly complex computation or optimization. Early work in *team theory* [12] sought to connect team decision making to game theory and optimization. Work in dependency structure matrices [13, 14] attempts to quantify important relationships in organizations and cluster or partition teams to optimize communication and work flow. Zakarian and Kusiak [15] developed a mathematical programming model to determine optimal team composition based on consumer requirements and product characteristics. Simulation approaches [16] have developed even more detailed models of the organization and intra-organizational communication supporting organizational engineering for project teams. Most recently, a variety of agent-based approaches have attempted to simulate complex innovation and decision interactions among different actors in an organization [17]. Although this review is necessarily cursory, it is interesting to note that many of the current approaches to computational organizational theory are characterized by the use of sophisticated simulation and optimization requiring the setting of myriad parameters in the hopes of learning something about one particular case.

In other fields, in earlier times, the tendency among engineers was to build and use simple models first. The field of structural mechanics is an interesting example. The simple model of cantilever beam bending, $\delta = \frac{PL^3}{3EI}$, where δ is beam end deflection, P is the point load at the beam end, L is the beam length, E is the modulus elasticity of the beam material, and I is the moment of inertia of the beam cross-section, is a simple gross model of beam response to load. Although the model is simple, it is reasonably accurate and, more importantly, gives a great deal of insight into the effect of different variables on beam response. Moreover, the simple model can be used to reason qualitatively about structural response of structures that do not identically fit the underlying assumptions of the model (constant cross-section, modulus, small deflection, etc.). Of course, structural engineers now also have access to detailed computer methods such as finite element methods (FEM) to give detailed predictions about structural performance, but engineers are still taught the little models first, and when reasoning about a structural problem they remain an invaluable tool even with FEM codes sitting on every structural engineer's desktop computer.

The idea of this paper is to go back to the future, and help resurrect an analytical way of thinking about organizations that strives to create and use little models of organizational interaction. Our immediate goal here is to create useful, analytical models that can be used to understand the balance between decision making and execution in teams. Longer term, we believe that a collection of such little models can be used for quantitative understanding and qualitative organizational insight, in much the same way that a structural engineer routinely turns to the simple models of the mechanics of materials. We are not alone in this effort, and recent studies [18, 19] inspired by thinking about organizations as ecologies have turned to simplified quantitative models to achieve this kind of insight. Huberman and Loch [19], for instance, examined the tradeoff between the benefits of group size on collaborative problem solving and the negative impact of larger groups on work motivation. Beckmann [20] presented a model of collaboration where productivity increases linearly with group size (as a result of collaboration), but only counterbalanced by a linear increase in time spent for communication (rather than working). This formulation resulted in a unique optimal team size.

Our viewpoint is that more of this sort of thing is necessary, and here we argue for stripping organizational analytics down to the barest possible bones. Of course, just as strength of materials style models coexist with the most sophisticated FEM codes, we expect the approach here to coexist with and complement more intricately detailed simulation and optimization approaches to organizational understanding. The benefits of quantitative and qualitative understanding of simplified models are indispensable in the physical domain, and we believe they will be just as indispensable in the organizational domain once enough of them are developed and methods are available for their integration and interpretation.

3 A Simple Model of Team Deciding and Doing

Teamwork in all its infinite varieties is enormously complex, involving the integration of multiple skills, different personalities, and different leadership styles to accomplish goals that may or may not be well specified or understood. Against this backdrop, it seems nearly impossible to make any progress toward a quantitative theory of teamwork until we recognize that teams usually must (1) decide something and (2) do something. If we decompose the problem of teamwork along these lines, we can create an interesting, yet simple, model of teamwork in fairly short order.

Consider a team of n members working on a task. We imagine that the team first discusses what is to be done with each of the n members taking T_1 time in the discussion. Thereafter, the task, requiring T_2 time units total is divided equally among the members of the team. The total time required for task completion can be written as follows:

$$T(n) = T_1 n + \frac{T_2}{n}. \tag{1}$$

Of course, we can imagine that the discussion may require more or less time depending upon the decision process involved. For example, a team led by a dictatorial leader would spend less time making a decision than a team that must reach consensus [5], and in a moment we will characterize that variability by permitting a more general power law term for decision making. Likewise, we can imagine that team doing might require more or less time than dictated by the assumption of equal task sharing if increased numbers encourage increased shirking or increase opportunities for work saving synergy. Here, too, it should be possible to extend the model in an elementary manner.

Examining equation 1 we note that the deciding term increases with increased team size and the doing term decreases with increased team size. It seems reasonable to expect a single, unique optimum, which can be determined by elementary means. Taking the derivative of equation 1 and setting the resulting expression to zero results in the time efficient or efficient team size n^* as follows:

$$n^* = \sqrt{\frac{T_2}{T_1}}. \tag{2}$$

Taking the derivative a second time establishes that the optimum is unique and a minimum for T_1 and T_2 greater than zero.

Examining a specific case is helpful, and Figure 1 shows the tradeoff when $T_2 = 10$ and $T_1 = 0.1$. The deciding and doing terms are plotted separately alongside the composite task time, and the minimum time occurs at the predicted value of $n^* = \sqrt{\frac{10}{0.1}} = 10$.

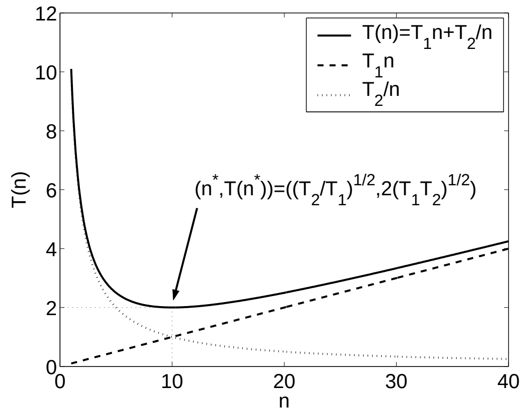

Figure 1. The solid line shows the overall task time in the simple team model of deciding and doing. As the theory predicts, the model has a unique minimum when the marginal costs of deciding and doing are balanced ($T_1 = 0.1$, $T_2 = 10$, $n^* = 10$).

The value of the minimal task time may be calculated by substituting equation 2 into equation 1:

$$T^* = 2\sqrt{T_1 T_2}. \tag{3}$$

In words, the efficient task time is twice the geometric mean of the individual deliberation time T_1 and doing time T_2.

One useful way to look at these results is in *dimensionless form*. Dividing equation 1 by equation 3 and letting $\tau = T/T^*$ yields the following:

$$\tau = \frac{1}{2}\left(\nu + \frac{1}{\nu}\right) \tag{4}$$

where $\nu = \frac{n}{n^*}$. Equation 4 is plotted in Figure 2.

Another useful way to look at these results may be borrowed from computer science [21]. In particular, the *speedup* may be defined as the ratio of time required by a single individual and the time required by the team. The speedup at arbitrary team size n may be calculated as

$$\mathcal{S} = \frac{T_1 + T_2}{T_1 n + \frac{T_2}{n}}. \tag{5}$$

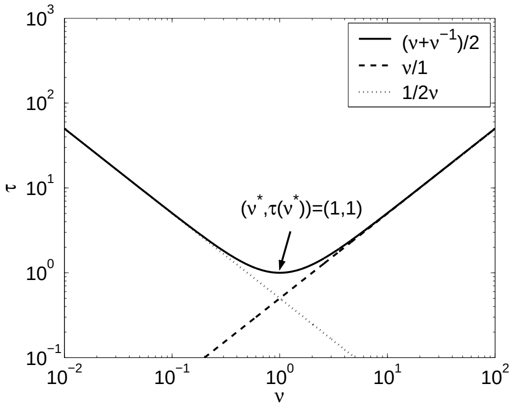

Figure 2. Dimensionless deciding-and-doing model.

The speedup $\mathcal{S}^*$ at optimal team size n^* may be calculated as

$$\mathcal{S}^* = \frac{T_1 + T_2}{2\sqrt{T_1 T_2}}. \tag{6}$$

In words, the optimal speedup is the ratio of the arithmetic mean of the task and deliberation time values divided by their geometric mean (Figure 3).

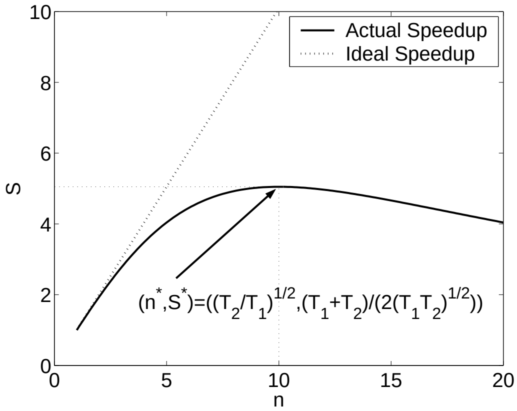

Figure 3. Actual speedup versus ideal speedup ($T_1 = 0.1, T_2 = 10, n^* = 10, \mathcal{S}^* = 5.05$).

4 Concern for Efficiency and Decision Quality

In business, time is money, and in the previous section we emphasized the importance of speed to the exclusion of other concerns; however, this limitation is not a failing of our simplified modeling approach, and in this section, we augment the model of the previous section with a concern for solution quality.

4.1 A simple model of decision quality

To do this, imagine that the solutions proposed by the individual team members are each assumed to be generated by independent stochastic trials, and each has success probability p. The overall success of the team is the probability of having at least one success among the n team member trials. Calling this probability the decision quality Q, we may calculate it in an elementary manner as follows:

$$Q = 1 - (1-p)^n. \tag{7}$$

In other words, the probability of having at least one success in n trials is the complement of having no successes. For convenience, we will call the quantity $1-Q$ the solution or decision error ε.

As can be seen in figure 4, solution quality increases monotonically with increased n, and solution quality is much better than a coin toss for values of $n > p^{-1}$.

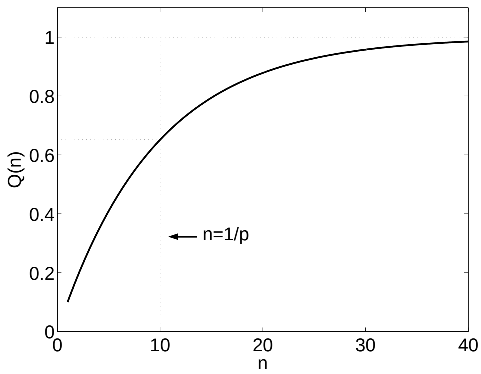

Figure 4. Solution quality improves with increased team size irrespective of deciding or doing time. In this particular case, $p = 0.1$ and the success probability is relatively high for values of $n > 1/p$ or 10.

By itself, this model is interesting enough, but we can examine the combined effect of time and quality on a single curve, and these can be compared in relation to the efficient team size of the previous section.

Here we consider the variation of the time required according to equation 1 and the solution quality of equation 7. Varying n from one to n^* and to a value much greater than n^* yields a curve of the general shape of figure 5.

At values of team size lower than n^*, T is higher than the minimum value, and the solution error is still decreasing as n increases. As a result, all points on the dashed portion of the T-ε curve are *dominated* by the point when $n = n^*$. When $n > n^*$, T increases and the error decreases. Thus, the solid portion of the curve represents a true tradeoff between solution speed and quality; a reasonable manager might choose to have larger teams to get improved quality.

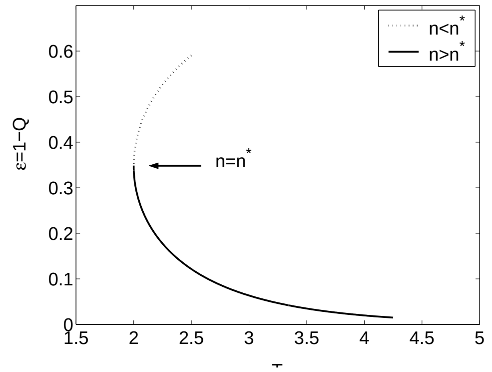

Figure 5. The relationship between time and quality for a team working on a given task. A team smaller than size n^* is not preferred because a long time is needed but the quality is even lower. A team larger than n^* achieves a higher quality by consuming a longer time.

4.2 Unequal numbers of decision makers and doers

The previous section suggests a practical means of obtaining both a desired level of decision quality and a reduction of the overall project time. In many teams, the decision-making group may be either larger or smaller than the task group. This suggests simple modifications to the basic model. Define the ratio of the deciding group to that of the task group as ρ. The time model (figure 1) can be expressed as

$$T = \rho T_1 n + \frac{T_2}{n}. \quad (8)$$

Since the number of decision makers is now ρn, the quality of decision may be modified accordingly:

$$Q = 1 - (1-p)^{\rho n}. \quad (9)$$

Here we would like to set decision time and quality independently and the introduction of different group sizes permits us to do just that. For a specified quality, say Q_s, we solve for the decision group size $n_d = \rho n$ as follows:

$$n_d = \frac{\ln(1-Q_s)}{\ln(1-p)}. \quad (10)$$

Then, given a desired project duration of T_s as long as the project duration is less than the decision time $n_d T_1$, we may calculate the size of task group required as follows:

$$n = \frac{T_2}{T_s - T_1 n_d}. \quad (11)$$

Dividing the team into subgroups eliminates waste in decision and execution, thereby giving a solution of specified quality in specified time.

5 Nonlinearities in Decision and Work Sharing

The basic model considers decision making as a process that grows linearly with team size; it considers execution as a process that decreases inversely with team size. Are these the only cases of interest, and, if not, can the model be extended to incorporate other situations? In this section we consider power law extensions to both terms in the basic model and briefly consider the rationale for so doing.

5.1 The complexity of decision making

We started by assuming linear complexity of decision making for simplicity, but there is nothing magical about linear growth. It has been long recognized that the number of pairwise relationships is an important quantity in understanding organizational structure [22]. The number of pairwise relationships is $\binom{n}{2} = O(n^2)$, and a decision process that permitted all pairwise conversations to take place would have a decision time proportional to the square of the team size $T_{deciding} = T_1 n^2$. More generally, the deciding time can be modeled as follows:

$$T_{deciding} = T_1 n^{c_1}, \quad (12)$$

where increasing c_1 increases the number of interactions required in the decision-making process. The case $c_1 = 0$ corresponds to a decision time that is constant irrespective of the size of the team, and the case $c_1 = 1$ corresponds to the base model.

5.2 The complexity of work sharing

Likewise, suppose that the work is not equally shared as was previously assumed. There are two cases of interest. The addition of more workers causes other workers to work more slowly, thereby increasing shirking or freeloading, or the addition of more workers inspires the existing workers to work harder than before, and there is synergy in numbers. Either way, a power-law relationship is a useful starting point for the modified doing time:

$$T_{doing} = \frac{T_2}{n^{c_2}}, \quad (13)$$

where $c_2 \geq 0$. The case $c_2 = 0$ corresponds to intricate craft work that cannot be speeded with additional help. The case $c_2 = 1$ is ideal work sharing as assumed in the basic model. The situation with $0 < c_2 < 1$ corresponds to those cases where the addition of workers does not reduce the execution time inversely with the number added. Cases with $c_2 > 1$ correspond to situations where the addition of workers promotes faster times that would occur if the work were split among n workers equally.

5.3 A power-law model of deciding and doing

Finally, a more general, power-law model of deciding and doing is created as the sum of equations 13 and 12:

$$T(n) = T_1 n^{c_1} + \frac{T_2}{n^{c_2}}. \tag{14}$$

Again, taking the derivative with respect to n and setting to zero, we recognize that $T(n)$ has a minimum at

$$n^* = \left(\frac{c_2 T_2}{c_1 T_1}\right)^{\frac{1}{c_1+c_2}}. \tag{15}$$

Note that for $c_1 = c_2 = 1$ that equation 15 reduces to equation 2 as it must.

For other values of c_1 such that $c_2 T_2 \geq c_1 T_1$ (a necessary condition for $n^* \geq 1$), the efficient team size n^* is a monotonically decreasing function of c_1 (which can be proved by showing that $\frac{\partial(\ln n^*)}{\partial c_1} < 0$ for $c_2 T_2 \geq c_1 T_1$). This suggests that as the complexity of decision making increases, the efficient team size decreases. Likewise, as c_2 increases, the efficient team size increases monotonically; this suggests that other things being equal, less shirking or more synergy reduces the efficient team size.

The optimal decision time may be calculated as follows:

$$T^* = \left[\frac{c_1 + c_2}{c_1^{\gamma} c_2^{1-\gamma}}\right] T_1^{1-\gamma} T_2^{\gamma}, \tag{16}$$

where $\gamma = \frac{c_1}{c_1+c_2}$. Dividing equation 14 through by equation 16 yields the dimensionless form

$$\tau = \frac{c_2 \nu^{c_1} + c_1 \nu^{-c_2}}{c_1 + c_2}, \tag{17}$$

where $\tau = T/T*$ and $\nu = \frac{n}{n^*}$ as before.

6 Elementary Optimization Problems More Generally

The foregoing analysis optimized team project time using straightforward elementary optimization, but this kind of analysis is not new. For example, the well-known *economic order quantity* or EOQ [23] is an example of this sort of thing, having exactly the mathematical form of equation 1, even though inventory and teamwork are intellectually distant.

Given the widespread use of equations such as EOQ, we believe that the systematic bounding analysis of team size, span of control, and other critical quantities in organizational theory, can result in a level of quantitative and qualitative insight in organizational theory as has been useful elsewhere. To that end, in this section we generalize the style of analysis of this paper in what we call an *elementary optimization problem*. In particular, we define the characteristics of an EOP, we give a few EOP exemplars, and then suggest how auxiliary functions and patchquilt analysis [24] can be used to extend the reach of EOPs beyond organizational problems that are merely elementary.

6.1 Defining an EOP

For this study, we say a function is an elementary optimization problem or EOP if it satisfies three characteristics:

1. An EOP is a function of one variable.
2. An EOP is the sum of a monotonically increasing function and a monotonically decreasing function.
3. An EOP is twice differentiable.

An EOP can be formalized as follows.

Definition 1 *An EOP can be written as*

$$\Phi(n) = f(n) + g(n), \tag{18}$$

where for n within some domain of interest $\mathbb{D}$, *(1)* $\frac{df}{dn} > 0$, *(2)* $\frac{dg}{dn} < 0$, *and (3)* $\frac{d^2\Phi}{dn^2}$ *exists.*

Note that the functions for $T(n)$ given in equations 1 and 14 satisfy the EOP properties.

The above definition of EOP can be extended to multiplicative form by the following theorem.

Theorem 1 *Given a function* $\Psi(n) = f(n)g(n)$, *where for* $n \in \mathbb{D}$, $f > 0$, $g > 0$, $\frac{df}{dn} > 0$, $\frac{dg}{dn} < 0$, *and* $\frac{d^2\Psi}{dn^2}$ *exists,* $\log \Psi(n)$ *is an EOP in domain* $\mathbb{D}$.

The theorem can be easily proven by taking logarithm on both sides of $\Psi(n) = f(n)g(n)$. Moreover, the optima of an EOP can be either explicitly expressible or implicitly expressible, which suggests the following definition.

Definition 2 *An EOP is called an explicit EOP (XEOP) if $\frac{d\Phi}{dn} = 0$ can be solved as an explicit function of the independent variable n; it is called an implicit EOP (IEOP) otherwise.*

Moreover, we define a simple EOP as follows.

Definition 3 *An EOP is simple if it has a single optimum in the domain of interest $\mathbb{D}$.*

It is worth to note that if $\Phi_1(n)$ is an EOP, $\Phi_2(n) = \Phi_1(h(n))$ is also an EOP in some other domain where h is invertible and monotonic. This is formally addressed in the following theorem.

Theorem 2 *Given that $\Phi_1(n)$ is a simple EOP in $\mathbb{D}_1$, for any $h(n)$ invertible, twice differentiable, and monotonic in $\mathbb{D}_2$, where $\mathbb{D}_2 = \{n|h(n) \in \mathbb{D}_1\}$, $\Phi_2(n) = \Phi_1(h(n))$ is a simple EOP in $\mathbb{D}_2$*

Proof. Assume that $\Phi_1(n) = f(n) + g(n)$, so $\Phi_2(n) = f(h(n)) + g(h(n))$.

1. $\Phi_2(n)$ is a function of one variable, n.

2. For $n \in \mathbb{D}_2$, $h(n) \in \mathbb{D}_1$. Because $h(n)$ is monotonic in $\mathbb{D}_2$, one of $f(h(n))$ and $g(h(n))$ is monotonically increasing, and the other is monotonically decreasing.

3. Φ_2 is twice differentiable in $\mathbb{D}_2$ because Φ_1 is twice differentiable in $\mathbb{D}_1$, and h is twice differentiable in $\mathbb{D}_2$. Specifically, $\Phi_2''(n) = \Phi_1''(h(n))\,(h'(n))^2 + \Phi_1'(h(n))h''(n)$.

4. $\Phi_2'(n) = \Phi_1'(h(n))h'(n)$. Because $h(n)$ is monotonic, $h'(n) \neq 0$. Solving $\Phi_2'(n) = 0$ is the same as solving $\Phi_1'(h(n)) = 0$. Assume that $\Phi_1(n)$ has an unique analytical optimum at n_1^*. $\Phi_2(n)$ has an unique analytical optimum at $n_2^* = h^{-1}(n_1^*)$.

Therefore, $\Phi_2(n)$ is a simple EOP in $\mathbb{D}_2$.

By Theorem 2, it is easy to extend EOP to other classes of functions from known EOPs. For instance, it has been shown in previous sections that $a_1 n^{b_1} + a_2 n^{-b_2}$ is an EOP. Therefore, $a_1 e^{b_1 n} + a_2 e^{-b_2 n}$ (exponential) and $a_1 \log^{b_1} n + a_2 \log^{-b_2} n$ (logarithm power-law) are EOPs. By keeping doing this, $a_1 e^{b_1 (n^c)} + a_2 e^{-b_2 (n^c)}$ is also an EOP and so on. The optima, optimal solution values, and dimensionless forms for a number of potentially useful simple explicit EOPs are listed in Table 1.

f = Power Law and g=Power Law

Mathematic Form	$\Phi(x) = a_1 x^{b_1} + \frac{a_2}{x^{b_2}}$
Optima	$x^* = \left(\frac{a_2 b_2}{a_1 b_1}\right)^{\frac{1}{b_1+b_2}}$
Optimal solution values	$\Phi(x^*) = \left(\frac{b_1+b_2}{b_1^{\gamma} b_2^{1-\gamma}}\right) a_1^{1-\gamma} a_2^{\gamma}$, where $\gamma = \frac{b_1}{b_1+b_2}$.
Dimensionless forms	$\tau = \frac{b_2 \nu^{b_1} + b_1 \nu^{-b_2}}{b_1+b_2}$

f = exponential and g=exponential

Mathematic Form	$\Phi(x) = a_1 e^{b_1 n} + a_2 e^{-b_2 n}$
Optima	$x^* = \frac{1}{b_1+b_2} \ln\left(\frac{a_2 b_2}{a_1 b_1}\right)$
Optimal solution values	$\Phi(x^*) = a_1^{1-\gamma} a_2^{\gamma} \left[\left(\frac{b_1+b_2}{b_1^{\gamma} b_2^{1-\gamma}}\right)\right]$, where $\gamma = \frac{b_1}{b_1+b_2}$.
Dimensionless forms	$\tau = \frac{b_2 e^{b_1 \nu} + b_1 e^{-b_2 \nu}}{b_1+b_2}$.

f = power law of an invertible function and g=power law of the same function

Mathematic Form	$\Phi(x) = a_1 (h(x))^{b_1} + a_2 (h(x))^{-b_2}$, where h is invertible and monotonic.
Optima	$x^* = h^{-1}\left(\left(\frac{a_2 b_2}{a_1 b_1}\right)^{\frac{1}{b_1+b_2}}\right)$.
Optimal solution values	$\Phi(x^*) = \left(\frac{b_1+b_2}{b_1^{\gamma} b_2^{1-\gamma}}\right) a_1^{1-\gamma} a_2^{\gamma}$, , where $\gamma = \frac{b_1}{b_1+b_2}$.
Dimensionless forms	$\tau = \frac{b_2 (h(\nu))^{b_1} + b_1 (h(\nu))^{-b_2}}{b_1+b_2}$, for $h(cx) = ch(x)$.

Table 1. The optima, optimal solution values, and dimensionless forms for a number of potentially useful EOPs.

6.2 Integrating multiple EOPs: A patchquilt approach

The derivation of simple models from straightforward reasoning and EOPs holds promise for the derivation of a number of models in organizational theory, but the complexity of organizations suggests that any truth captured by a single model will be somewhat localized. To build up a more realistic model of organizational performance will require the integration of different models, and here a technique discussed in some detail elsewhere [24] may be useful.

In particular, the use of dimensional analysis [25] may be used to assemble a patchquilt of different models in a rational manner. Suppose, for example, that we have multiple models of efficient team size derived under different assumptions. In one model the team size is independent of organizational size m, that is, $n^* = n_0$, a constant. In the other model, n^* goes up as the square root of the organizational size, $n^* = c\sqrt{m}$. We could draw both curves on a graph of n^* versus m (Figure 6). Which curve applies when?

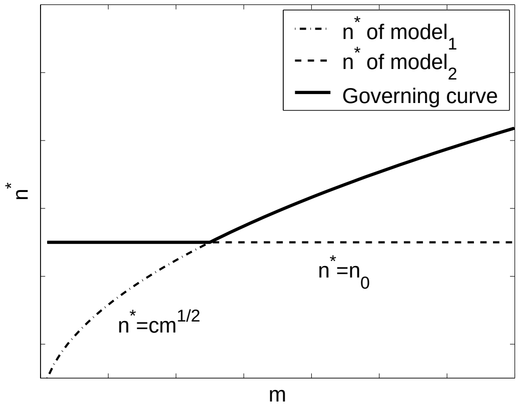

Figure 6. The optimal team sizes n^* of two different models.

If both models were derived properly, it is reasonable to believe that both models will govern performance asymptotically in some portion of the team-organization space we are examining. For example, in small organizations, we might reason that the efficient team size model of this paper might govern up to some critical organizational size, and we might look for other models to govern as organizational size grows. If this is the case, then we might expect the $n^* =$ constant model to govern at low values at m and the square root model to govern at higher values. We can build a *patchquilt model* integrating them both by simply following one model on one side of their intersection and one model on the other side of the intersection as shown in the figure.

This approach to problem solving is routine in fluid mechanics and it has proven extremely useful more recently in understanding and designing effective genetic algorithms [24]. Work is now ongoing at Illinois to integrate a number of different organizational models along exactly these lines.

7 Conclusions

This paper has derived a number of models of efficient team size (ETS) by considering the tradeoff between deciding and doing. A basic model assuming linear complexity of decision and inversely linear improvement in execution time results in an efficient team size that goes as the square root of the overall task time divided by an individual's decision time. The basic model has been augmented through the introduction of an auxiliary equation for solution quality and the size-quality tradeoff has been explicitly examined. The basic model has been extended by introduction of power-law models for decision and execution, the idea of a more general class of elementary optimization problems or EOPs has been considered, and a number of EOPs have been enumerated and solved.

There has been progress in understanding organizational theory computationally in recent years, but the busy complexity and one-off nature of those modeling efforts makes us wonder whether there aren't simpler analytical models to help guide our organizational thinking and design. This paper has offered up a number of such models and an approach to deriving and integrating more of them. Although we are unsure how far this methodology can be pushed, we are hopeful that it can shed important light on a difficult, yet important, area of concern.

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