

A Predictive Model for New-Venture-Based Region Growth

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Abstract

Many communities have taken interest in developing technology companies in their region. Technology companies are known to bring higher average salaries, skilled workers, and increases in tax base. In this paper, we develop an analytical method to predict how a set of new ventures collectively grow and what specific effects this may have on a local region economy. In particular, we provide a model to predict growth variables such as job creation, and a method to calculate ROI for investors (ROII) and economy (ROIE). The suggested ROI calculation method is based on the proposed venture growth model, which requires two inputs. The first is a statistical characterization of how companies grow. The second input is a function which characterizes the resources invested over time for creating new businesses, referred to as Investment Profile (IP). The paper also includes a realistic example that compares the effects of different investment profiles. That is, it compares the long term effects of launching 1, 5, 10, 15, and 20 companies per year for 15 years sequentially. The results are provided in terms of job creation, ROII and ROIE with a wide range of sensitivities and confidence intervals. From the case, we find that not only the ROII is potentially substantial, but also an even larger return is made to the economy (ROIE) through the effects of long term job creation.

Keywords: Entrepreneurial Engineering, Entrepreneurial Finance, Economic Growth, Quantitative analysis, Geographic Region.

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Nomenclature

Variables used in core model

$X_i(t)$:	Number of employees in company i within the portfolio in year t after inception.
$\mu(t), \sigma(t)$:	Company growth characteristics, mean and standard deviation of $X_i(t)$ in year t - also referred to as MIR and DIR.
$g(t)$:	Investment profile (IP), number of companies launched in year t of the analysis
$X_p(t)$:	Portfolio size in year t - Number of employees in the portfolio in year t
$\mu_p(t), \sigma_p(t)$:	Portfolio growth characteristics, mean and standard deviation of $X_p(t)$ in year t
$\tilde{X}_p(z)$:	Lower bound on Portfolio size with probability $\Phi(z)$
z :	Standard normal variable. $z = 0, 1, 2, 3$
$\Phi(z)$:	$\Phi(z)$ is the lower tail area of the standard normal distribution at a specific z value.
ROII:	Return on Investment (ROI) to investors for the Portfolio
ROIE:	Return on Investment (ROI) to the economy (i.e. local community) of the Portfolio
M:	Money circulation multiplier within an economy (Economic Multiplier)
s :	Average salary per employee
J_i :	Jobs created per investment
V_E :	Company value per employee
I:	Investment per company
F_{ROII} :	Conversion factor between 'jobs per investment' (J_i) and ROI to investors (ROII).
F_{ROIE} :	Conversion factor between 'jobs per investment' (J_i) and ROI to the economy (ROIE)
F_{shape} :	A second conversion factor to obtain ROIE from 'jobs per investment' (J_i) based on Portfolio growth shape.

Variables used in the approximation model

W:	Investment time window
n^o :	Fixed number of new ventures started in a region per year for W years
t_μ^o :	Time for the mean number of employees in a company to reach a steady state level
t_σ^o :	Time for the standard deviation of number of employees in a company to reach steady state
t^o :	$\max(t_\mu^o, t_\sigma^o)$
μ_{max} :	The steady state (constant) mean number of employees in a company
σ_{max} :	The steady state (constant) standard deviation for the number of employees in a company

1. Introduction

Policy makers often strive to grow high technology companies in their local regions in order to bring higher average salaries, skilled workers, and increases in tax base. Studies to aid the economic development policy have so far focused on understanding the characteristics of a region that is likely to attract and grow new businesses. Some of these studies have even attempted to list the resources that a community can offer to aid business growth (Buss, 1997; Ghanem, 1997). Studies like these are very useful to policy makers who are interested in creating growth in their local regions. However, no studies currently exist which help the policy maker predict the potential benefits to the local

economy over time or correlate them to the amounts of investment, which would be required to initiate these patterns of growth. To ascertain the returns from these investments, we first need to know the answers to two fundamental questions: First, which types of companies are most likely to succeed within that geographic region? Second, how will a region grow based on specific investments in new technology companies? We addressed the first question in (Shariff et al., 2003). In this paper we focus on the latter question by developing an analytical method to predict how companies grow and what specific effects this may have on a local region.

We begin by defining the concept of a *Portfolio* and illustrating the elements of our model. We narrowly define the concept of a geographic portfolio (or just “*Portfolio*”) to be a set of companies that are growing within a particular geographic region because we are primarily interested in economic growth within a geographic region. Next, we define functions to statistically characterize how a company might grow over time – which represents a fundamental input to the model. We define the Mean Impulse Response and Deviation Impulse response functions (MIR and DIR) to characterize company size measured in number of employees in every year after the creation of a sample company. Although our formulation is general, MIR and DIR can be easily parameterized for various company types such as “marginal / slow growth” or “promising / high growth” companies. In the form of an example, we provide realistic parameterizations for each of these growth models based on well known “characteristics” or “rules of thumb”. Finally, we introduce the concept of an Investment Profile (IP). The investment profile will define how many new growth companies can be launched within a region limited by the availability of both resources as well as suitable investment opportunities.

In this paper, we show that the statistical characteristics for a *Portfolio* of companies, over long periods of time, can be attained by convolving the “impulse response” functions with the “investment profile”. These characteristics are important in understanding and predicting the effects of growth when considering a geographic region. Using principals of finance, we are able to quantify the risk and the returns achieved due to various investment profiles. We seek to understand the Return on Investment (ROI) to the community, not just to the investors.

In the next section, we present a short literature review. In Section 3, we develop a model of Portfolio growth. In Section 4, we provide formulations to obtain the ROI for investors (ROII) and for the economy (ROIE) based on various investment profiles (IP). Section 5 presents useful approximations of Portfolio growth and ROI using simple, but typical forms of MIR, DIR, and IP. Section 6 provides a realistic case study which compares the impact of different investment profiles. Finally, in Section 7, we summarize our results regarding diversification gains and sensitivities for ROII and ROIE.

2. Literature Review

This paper is about entrepreneurial finance issues for a *Portfolio* of companies within a specific geographic region. Literature in this area is fundamentally at the intersection of entrepreneurship, finance, and economics.

In some sense, this work would fall under the area of entrepreneurship. However, most entrepreneurial research has been done from the perspective of growing a single company. Typical studies of this type describe characteristics of entrepreneurs (Moore, 1986; Roberts, 1991) or an entrepreneurial process (Singer, 1995; Timmons, 2001; Bhide, 2000). Moore attributes the ability to rapidly make strategic changes in relatively short time as an aspect of entrepreneurial behavior.

In a narrower sense, we are concerned with financial aspects of the entrepreneurial process. Denis (2003) and Cardullo (1999) provide an overview of growing entrepreneurial finance literature. While Denis has a more general overview of the different aspects of entrepreneurial financing, Cardullo more specifically examines the technology companies, in terms of enterprise formation, financing and growth. Studies within the field of entrepreneurial finance have mostly been about:

- The financing issues of creating a new venture (Carayannis et al., 1997; Carrie`re et al., 1996; Jacobus, 1997) including attraction of Venture Capital (VC), and
- The perspective of the Venture Capital investor (Denis, 2003; Kanninen, 2003).

The former is concerned with term sheets, methods for negotiating valuation, options in investing and financing new technologies in entrepreneurial firms etc, while the latter is concerned with the inter-dependencies of investing in multiple opportunities

at the same time. While the VC perspective may address prediction models for growth, neither of these cases addresses the implications to the local economy.

In the narrowest sense, we might consider the intersection of entrepreneurial finance and economic issues. Economic growth has been studied at many levels, including the national (Barro et al., 1991), the regional (Barro et al., 1992), and the local (Glaeser et al., 2001) (i.e., at the level of the agglomeration). For example, many scholars have been interested in the growth of new economies in the Silicon Valley style agglomeration of industries (Porter, 1990; Krugman, 1991; Ellison et al., 1999). Ellison et al. (1999) qualitatively discusses examples where clustering effect of companies has been pronounced. Previous work has studied in depth the growth of these new economies.

We are not presently aware of any study, which seeks to relate investments in new business creation to actual growth metrics such as jobs created, changes in local economic metrics, ROI for the Investor, and ROI for the economy. In this paper, we seek to accomplish precisely these results through general formulation.

3. A Model for Predicting Portfolio Growth

In this section, we develop an analytical model which can be to predict how sets of companies grow and what specific effects this may have on a local region. In particular, this model has two types of input, as shown in Figure 1. The first input is a characterization of company growth. The second is the resources invested over time for creating new businesses. The model can be used to predict growth variables including job creation, ROI for investors, and ROI for the community. The rest of this section describes elements of the proposed model in detail.

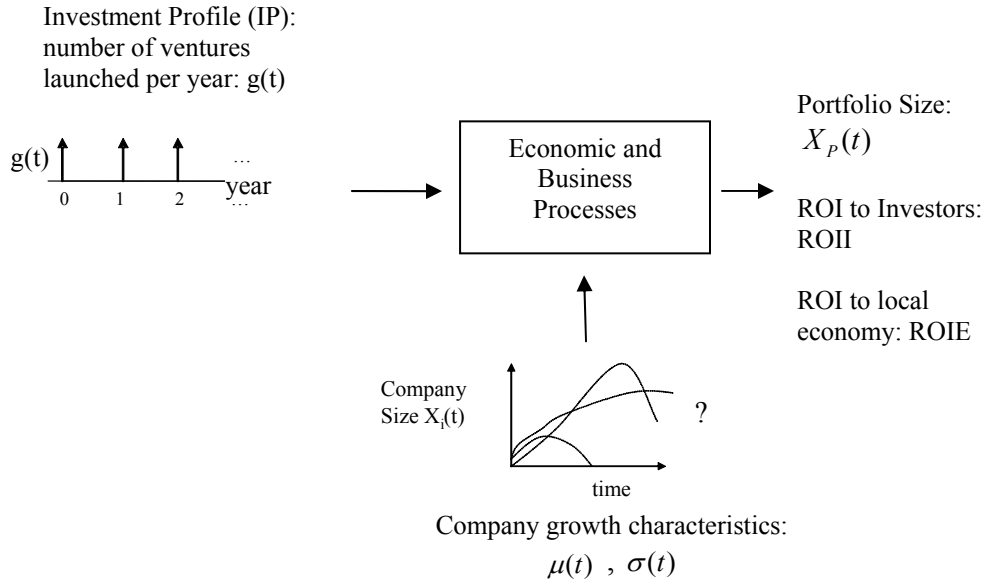


Figure 1: Elements of the Proposed Growth Model

A. Portfolio

In this model, a *Portfolio* is defined as a collection of companies started in a region. The Portfolio is measured by total number of employees $X_p(t)$ at time t since creation of the first company in the Portfolio. The number of employees in each individual company within the Portfolio are expressed as $X_i(t)$ for company i . Specifically, $X_i(t)$ is a random variable specifying the number of employees for company i in year t after its inception. Note that we assume all the random variables $X_i(t)$ to be identically distributed and independent across all companies i in the *Portfolio*. By making this simplifying assumption, we do not consider the environmental effects like general economic prosperity, which might affect all companies in the *Portfolio* in a similar or correlated manner.

In this model, “company growth” is measured strictly by the increase in number of employees in the company. To be precise, $\mu(t)$ is the mean and $\sigma(t)$ is the standard deviation of the random variable $X_i(t)$. For an individual company, we define:

1. The Mean Impulse Response (MIR): $\mu(t)$ gives the average company size in employees for year t after the creation of a typical company where for $t \geq 0$.
2. The Deviation Impulse Response (DIR): $\sigma(t)$ gives the standard deviation of employees in a typical company, also parameterized by t years after company creation.

Previous studies have shown that companies can be categorized as marginal versus promising companies and that the category of company being launched will be a factor to parameterize $\mu(t)$ and $\sigma(t)$.¹

B. Investment Profile(IP)

An investment profile (IP) describes how many companies can be started over the course of future years. The limits of starting new companies within the *Portfolio* may include funding, number of incubation stage companies from which to choose, or other resources that the community may need to provide. The net effect is that the local region and economy are limited in terms of the number of companies that can be incubated at a given time.

We define an investment profile (IP) as a function $g(t)$, which is the number of companies enabled by investment in year t .² We propose constructing an IP from impulse functions. The impulse function $\delta(t - t_0)$ represents the policy of creating one company in year t_0 . If one company is started every year in a region, this IP can be written as follows, where “ a ” is a dummy variable used to index the IP $g(t)$.

$$g(t) = \sum_{a=0}^{\infty} \delta(t - a) \quad (1)$$

In general, if θ companies are launched every year, the policy can then be expressed as:

$$g(t) = \theta \sum_{a=0}^{\infty} \delta(t - a) \quad (2)$$

¹ We define high growth companies to be worthy of professional investment, i.e. Venture Capital etc. We chose to focus our analysis on high growth companies because the capacity of a region to sustain marginal growth companies is limited by the existence of money circulation driven by government institutions and larger companies which operate over larger geographic regions. Many marginal growth companies provide only local services.

² For example $g(t) = [1,1,1,1]$ means that one company will be started each year for the next 4 years.

Of course, $g(t)$, in general, can take the form of any arbitrary function.

C. Portfolio Growth Prediction

We seek to predict the growth pattern of the *Portfolio* based on any given mean response function (MIR), deviation response function (DIR), and investment profile (IP). In particular, we show that the mean number of employees in the *Portfolio* can be calculated by convolving the MIR and the IP. Additionally, we show that the variance of the number of employees in the *Portfolio* can also be calculated by a similar method.

Let $g(t)$ be a particular investment profile (IP). Figure 2 shows a forecasting table for the number of employees in the *Portfolio*. In this example, $g(t) = [1, 2, 1\dots]$. Also, as implied by $g(0) = 1$, there is one company launched at time zero and it will have $X_1(1), X_1(2), X_1(3)$, etc. ... employees in the company’s first, second, third, etc. ... years of operation respectively.

Years	1	2	3	4	5	6	7	8
$g(0)=1$	$X_1(1)$	$X_1(2)$	$X_1(3)$	$X_1(4)$	$X_1(5)$	$X_1(6)$	$X_1(7)$	$X_1(8)$...
$g(1)=2$		$X_2(1)$	$X_2(2)$	$X_2(3)$	$X_2(4)$	$X_2(5)$	$X_2(6)$	$X_2(7)$...
		$X_3(1)$	$X_3(2)$	$X_3(3)$	$X_3(4)$	$X_3(5)$	$X_3(6)$	$X_3(7)$...
$g(2)=1$			$X_4(1)$	$X_4(2)$	$X_4(3)$	$X_4(4)$	$X_4(5)$	$X_4(6)$...
...									
...									

Figure 2: Forecasting Table

With each company that is added to the *Portfolio*, new jobs are created, and hence the number of employees within that *Portfolio* increases. In the next year, there are two companies launched as implied by $g(1) = 2$. $X_2(1), X_2(2), X_2(3)\dots$ represent the number of employees in company 2 and are also shown staggered to the right by one year to represent that they were started with one year later than company 1. $X_3(1), X_3(2), X_3(3)\dots$ represent the number of employees in company 3 and are interpreted in a similar manner.

In general, we can calculate the total number of employees in each year for the *Portfolio* by taking the sum down any given column of the table - as in the equation below.³

$$X_p(t) = \sum_{a=0}^{\infty} \sum_{b=1}^{g(a)} X_i(t-a) \quad (3)$$

From this relation we can see that the expected value and variance of the number of employees in the *Portfolio* is the convolution of the mean and variance response functions with the investment profile (IP).

$$\mu_p(t) = E[X_p(t)] = E\left[\sum_{a=0}^{\infty} \sum_{b=1}^{g(a)} X_i(t-a)\right] = \sum_{a=0}^{\infty} \sum_{b=1}^{g(a)} E[X_i(t-a)] = \sum_{a=0}^{\infty} \sum_{b=1}^{g(a)} \mu(t-a) = \sum_{a=0}^{\infty} \mu(t-a) g(a) \quad (4)$$

Similarly,

$$\sigma_p^2(t) = VAR[X_p(t)] = VAR\left[\sum_{a=0}^{\infty} \sum_{b=1}^{g(a)} X_i(t-a)\right] = \sum_{a=0}^{\infty} \sum_{b=1}^{g(a)} VAR[X_i(t-a)] = \sum_{a=0}^{\infty} \sigma^2(t-a) g(a) \quad (5)$$

Note that this is true only because of our independence assumption. That is, $VAR[X_1+X_2] = 2 VAR[X]$ when X , X_1 , and X_2 are independent and identically distributed. Rewriting Equations (4) and (5) using the standard notation for convolution yields:

$$\mu_p(t) = \mu(t) * g(t) \quad (6)$$

$$\sigma_p^2(t) = \sigma^2(t) * g(t) \quad (7)$$

We should note that we assume the distribution of $X_p(t)$ to approach the Normal distribution. Recall that $X_p(t)$ is the weighted sum of independent random variables for any given value of t . Specifically, we may approximate the distribution as follows:

$$X_p(t) \sim N[\mu_p(t), \sigma_p(t)] \quad (8)$$

Once the mean and standard variation of the Portfolio size, $X_p(t)$, are determined by Equation (8), we can readily determine lower bounds on our estimates of the number of companies in the Portfolio, given a specific confidence level. We define the lower

³ The result would be the same if we indexed time (t) and employee counts $X_i(t)$ starting with $t=0$. We chose to illustrate the table with years starting with 1 only to match the simulation table in the Appendix.

bound to be $\tilde{X}_p(z)$ which specifies the minimum number of employees in the Portfolio with a probability of $\Phi(z)$.⁴

$$\tilde{X}_p(z) = \mu_p(t) - z\sigma_p(t) \tag{9}$$

Furthermore, plotting the mean and standard deviation of $X_p(t)$ using a technique of financial portfolio analysis [Markovitz, 1952] can also be very instructive as we will see in the case example. In Figure 3, we illustrate the concept of financial portfolio analysis by plotting the mean number of employees on the Y-axis representing a measure of how much the Portfolio has returned over time. On the X-axis, we plot the standard deviation of number of employees in the Portfolio as a measure of risk. The Portfolio at any point in time can be represented as a point in this space. Any point with a higher mean for the same deviation is a better choice. Similarly, any point with a lower deviation given the same mean is a better investment alternative. Any line in the first quadrant through the origin represents constant return to risk ratio. In general, points higher or to the left would be better investments (Perold, 1995).

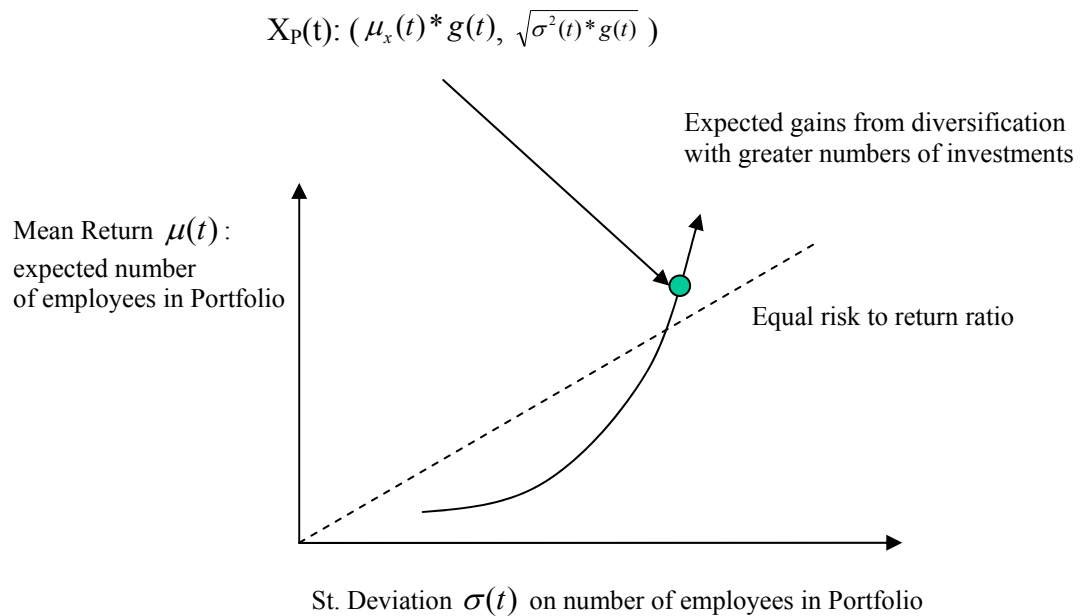


Figure 3: Financial Portfolio Analysis

⁴ For example, $z = 1$ means that with probability 84.1%, the number of employees in the Portfolio will be at least $\tilde{X}(1)$.

We expect that when the number of investments is small, that the deviation of outcomes will be high, but as the number of investments increases, the deviation becomes smaller accordingly relative to the mean. Simply stated, our risk will reduce relative to return as we launch more new ventures and diversify our investment. Furthermore, this reduction of risk (diversification gain) is likely to be diminishing as investments continue over time. In later analysis, we intend to parameterize and quantify these diversification gains.

4. ROI Estimation Method

One of our goals is to provide a simple mapping between the growth of the *Portfolio* and the returns to the investors and community. To do this, we rely on techniques that can map the intermediate variable of employees in the *Portfolio* to variables like company value and revenue, which are needed to determine ROI. We suggest the following generalized method to determine ROI to the investors and to the economy based on our choice of employee count as the driving intermediate variable:

1. Estimating *Portfolio* size: The first step is to construct a table for predicting *Portfolio* size (at particular time points) based on potential Investment Profiles (IPs). We use Equations (6) and (7) to calculate the mean and standard deviation of the *Portfolio* size, $X_P(t)$. Then, we construct confidence intervals around *Portfolio* size using a Gaussian approximation for the *Portfolio* size as suggested in Equations (8) and (9).

2. Calculate ROII:

By definition and without adjustment for the time value of money, ROI to investors (ROII) is the value of all companies in the portfolio divided by the total amount invested in all companies. The value of all companies, shown in the numerator of Equation (10), is the number of jobs created, multiplied by the value of each company per employee. The denominator includes a term for the number of total investments and a term for the investment per company. Typically, we would evaluate this ROII at a year T large enough that the steady state effects of the investments are evident and that the *Portfolio* size has effectively reached its peak.

$$ROII(T) = \frac{X_P(T) V_E}{\sum_{a=0}^{\infty} g(a) I} \quad (10)$$

In order to separate company growth characteristics from environmental characteristics (e.g. V_E and I), we prefer to represent ROI to investors (ROII) in a form which includes the multiplier F_{ROII} as a conversion factor between ‘jobs per investment’ (J_I) and (ROII).

$$ROII = J_I * F_{ROII} \quad (11)$$

Where, F_{ROII} is:

$$F_{ROII} = \frac{V_E}{I} \quad (12)$$

And, the long term jobs per investment (J_I) is formally characterized by the steady state of the Portfolio size, if it exists, divided by number of investments:

$$J_I = \frac{X_P(T)}{\sum_{a=0}^{\infty} g(a)} \quad (13)$$

3. Calculate ROIE:

Return on investment to the economy (ROIE) is more complex to estimate. For ROIE, we must consider the money circulation caused by the investments. In general, we suggest that the total salary of all employees in the portfolio be integrated over the growth of the portfolio. To understand the full impact of the increase in money circulation, we must also account for a multiplier effect which describes that these funds continue to circulate in the local economy.⁵ Without further complication from the time value of money, we may calculate the ROIE at time T as shown in Equation (14), below.

$$ROIE(T) = \frac{sM \sum_{t=0}^T X_P(t)}{I \sum_{a=0}^T g(a)} \quad (14)$$

⁵ There is a large amount of scholarly work that uses economic multipliers in studying or projecting economic growth. Kay lists examples of the application of multipliers in modern research in the area of economic policy decision-making (Kay, 2002). Another researcher has used multipliers in predicting economic growth from the growth in the agriculture in Ethiopia (Block, 1999).

Once again, we separate company growth variables from environmental variables (e.g. s and M) using a form which includes the conversion factors F_{ROIE} and F_{shape} .

$$ROIE(T) = M \cdot F_{ROIE} \cdot F_{shape} \cdot J_I \quad (15)$$

Where, F_{ROIE} and F_{shape} are:

$$F_{ROIE} = \frac{S}{I} \quad (16)$$

$$F_{shape} = \frac{\sum_{t=0}^T X_P(t)}{X_P(T)} \quad (17)$$

F_{shape} is a conversion factor which relates the area under the Portfolio growth function to the steady state (peak) value of the same function. Note that F_{shape} is small if the peak value of the Portfolio size is obtained very quickly.

In the next section we will derive approximations for the distribution of $X_P(t)$ (as in Equation (8)) and the corresponding approximations for ROII and ROIE of Equations (10) and (14).

5. Approximations and Long-Term Behavior

In this section, we introduce a piecewise linear approximation for the Mean and Deviation Impulse Response functions (MIR and DIR). Based on our empirical analysis, we show that such approximations are reasonable (see Appendix A). Using these approximations along with a “typical” IP, we develop closed form approximations for Portfolio characteristics.

We start by building a sample IP. A typical IP is to invest in some fixed number of new ventures (n^o) on an annual basis for a window of time (W years). In this case a typical $g(t)$ takes the form shown in Figure 4a. We approximate the MIR and DIR functions using piecewise linear segments as shown in Figure 4b and 4c based on substantial empirical evidence described in the Appendix.

In the MIR representation, we assume a linear growth phase from time 0 until year t^o_μ , after which we assume that the mean number of employees in a new firm stays

relatively stable at a maximum value of μ_{max} over a substantially longer period of time (i.e. to infinity in the model).

Likewise, for DIR, we assume a linear growth phase from time 0 until year t_{σ}^o , after which we assume that the standard deviation of number of employees in a new firm stays relatively stable at a maximum value of σ_{max} over a substantially longer period of time (i.e. to infinity in the model). Also note that t_{σ}^o and t_{μ}^o are not necessarily the same in value.

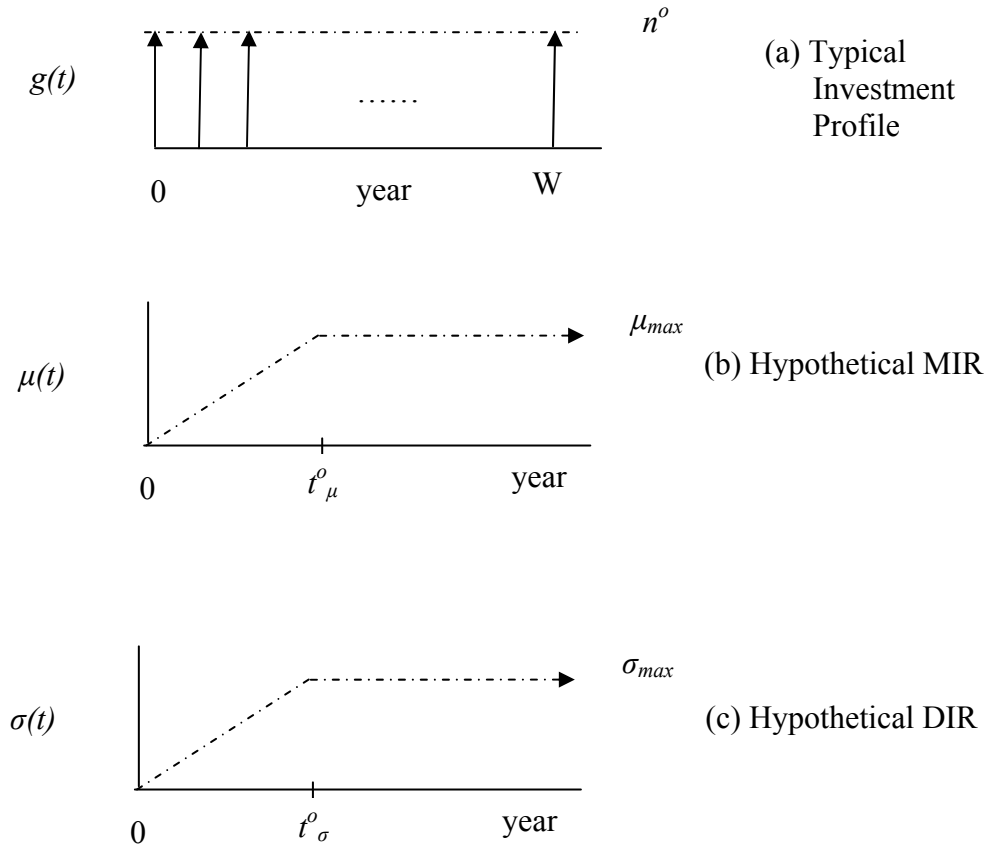


Figure 4: Approximate Functions for IP, MIR and DIR

Given these approximations, we are able to simplify the following symmetric closed form expressions for $\mu_P(t)$ and $\sigma_P(t)$:⁶

⁶ Note that we assume the investment window W is larger than the growth phase of one particular company $\max(t_{\sigma}^o, t_{\mu}^o)$.

$$\mu_p(t) = \begin{cases} \frac{1}{2} \mu_{\max} n^o t / t_{\mu}^o & \text{for } 0 \leq t \leq t_{\mu}^o \\ \mu_{\max} n^o (t - t_{\mu}^o) + \frac{1}{2} \mu_{\max} n^o \left[1 - \frac{t - W}{t_{\mu}^o}\right] & \text{for } t_{\mu}^o < t < t_{\mu}^o + W \\ \mu_{\max} n^o W & \text{for } t \geq t_{\mu}^o + W \end{cases} \quad (18)$$

$$\sigma^2_p(t) = \begin{cases} \frac{1}{2} \sigma_{\max}^2 n^o t / t_{\sigma}^o & \text{for } 0 \leq t \leq t_{\sigma}^o \\ \sigma_{\max}^2 n^o (t - t_{\sigma}^o) + \frac{1}{2} \sigma_{\max}^2 n^o \left[1 - \frac{t - W}{t_{\sigma}^o}\right] & \text{for } t_{\sigma}^o < t < t_{\sigma}^o + W \\ \sigma_{\max}^2 n^o W & \text{for } t \geq t_{\sigma}^o + W \end{cases} \quad (19)$$

Upon examination, the reader will note that we have effectively simplified the expression for the long term, steady state behavior of the Portfolio – that is after all investments have been made and after their effects have been realized, e.g. when $t > W + \max(t_{\mu}^o, t_{\sigma}^o)$. The mean and variance of the Portfolio size in employees simplifies to $(\mu_{\max} n^o W)$ and $(\sigma_{\max}^2 n^o W)$ respectively. As we would expect, both these terms are linearly dependant on $n^o W$ which is the total number of investments made (i.e. $\sum_{a=0}^{\infty} g(a) = n^o W$).

To leverage portfolio theory (Perold, 1995), we plot the steady state Portfolio on a risk versus reward field with the point P for large enough values of t (i.e. long-term). The coordinates of point P are shown in Equation (20). This provides us a closed form approximation for the point illustrated in Figure 3.

$$P(X_P(t)) = (\sigma_{\max} \sqrt{n^o W}, \mu_{\max} n^o W) \text{ for } t > W + \max(t_{\mu}^o, t_{\sigma}^o) \quad (20)$$

The lower bound $\tilde{X}_p(z)$, which specifies the minimum number of employees in the Portfolio with a probability of $\Phi(z)$ becomes:

$$\tilde{X}_p(z) = \mu_{\max} n^o W - z \sigma_{\max} \sqrt{n^o W} \quad (21)$$

By substituting Equation (21) into Equation (13), we obtain an estimate of J_1 (with confidence intervals).

$$J_I = \frac{\tilde{X}(z)}{n^o W} \quad (22)$$

Then, by substituting Equation (22) in (10) results in the following approximation for ROII with confidence intervals based on the steady state results:

$$\text{ROII}(z) = [\tilde{X}_p(z) / n^o W] [V_E / I] = \left(\mu_{\max} - z \frac{\sigma_{\max}}{\sqrt{n^o W}} \right) \frac{V_E}{I} \quad (23)$$

The second term $[V_E / I]$ represent the driving environmental variables which determines the ROI. Obviously, ROII increases linearly with value per employee (V_E) and inversely proportional to the investment per company (I). The maximum value of ROII is obtained for large numbers of investments ($n^o W$) which results in an ROII value of $\mu_{\max} \frac{V_E}{I}$. Moreover, the “diversification gain” to ROII exhibits diminishing rates of return as verified by the negative value of the second derivative of Equation (23).

To calculate an approximation for ROIE, we substitute into Equation (22) into Equation (15) results in:

$$\text{ROIE}(z) = \frac{\tilde{X}_p(z)}{n^o W} M \quad F_{\text{ROIE}} \quad F_{\text{shape}} \quad (24)$$

We know that by year $t^o = W + \max(t^o_\mu, t^o_\sigma)$, $\tilde{X}_p(z)$ has reached a steady state value of $\mu_{\max} n^o W - z \sigma_{\max} \sqrt{n^o W}$. If we assume a linear growth of employees in the Portfolio until t^o (which matches our case evaluations and trials in the appendix) then the salary paid out from time 0 to t^o is $s\tilde{X}_p(z)t^o/2$. For the remaining years, we discount the aggregated salary to be 10 times the 1 year peak return.⁷ The sum of these two areas is used in Equation (17) to derive a specific value of F_{shape} :

$$F_{\text{shape}} = \frac{s\tilde{X}_p(z)t^o/2}{\tilde{X}_p(z)} + \frac{10\tilde{X}_p(z)}{\tilde{X}_p(z)} = \frac{t^o}{2} + 10 \quad (25)$$

By substituting Equation (25) in (24), our approximation for ROIE becomes:

⁷ This is a common technique, similar to valuation of company with flat expected earnings with a P/E multiplier of 10.

$$\text{ROIE}(z) = \frac{s\tilde{X}_p(z)\left(\frac{t^o}{2} + 10\right)M}{n^oWI} = \frac{(\mu_{\max}n^oW - z\sigma_{\max}\sqrt{n^oW})\left(\frac{t^o}{2} + 10\right)sM}{n^oWI} \quad (26)$$

For ROIE, we can approximate that in the long run, every dollar invested returns ROIE(z) dollars with probability $\Phi(z)$. Similar diversification gains exist for ROIE as with ROII.

6. Case Study: Portfolio Growth based on Various Investment Profiles

In this section, we provide an example for analyzing *Portfolio* growth using the model presented in Sections 3-5. In particular, we test 5 different investment profiles for launching high growth companies:

- 1) One company launched every year for 15 years and then no more investments
- 2) Five companies launched every year for 15 years and then no more investments
- 3) Ten companies launched every year for 15 years and then no more investments
- 4) Fifteen companies launched every year for 15 years and then no more investments
- 5) Twenty companies launched every year for 15 years and then no more investments

6.1 Case Analysis of Portfolio Growth Characteristics

In order to use the model, we start by estimating the mean and standard deviation of the number of employees in the entire *Portfolio* given various investment profiles. The graphs in Figure 5 are obtained by convolving the mean and variance response functions (MIR and DIR provided in Appendix A) with each of the five above specified investment profiles. Our intention in this example is to predict the size and variability of the *Portfolio* during and after the end of 15 years of consistent investment in growth oriented companies. First we use the general formulation presented in Sections 3-4 and then we compare these results to the ones found by applying the approximation technique described in Section 5.

We have plotted the mean and standard deviation of the number of employees for all five investment profiles based on Equations (6) and (7). Each curve represents a different number of years into the investment process. For example, the “two year” curve has 5 points. The five points on this curve (and all other curves) give the statistics for

the number of employees in the *Portfolio*. The point on each curve specifies statistics depending on whether 1, 5, 10, 15 or 20 companies were launched each year. Statistics for 1 company per year is the point being closest to the origin and 20 companies per year point being farthest away – on each curve. For example, if the reader is to examine the 3 year curve and the 4th point away from the origin on that curve, it specifies that if 15 companies are launched yearly, then we expect the number of employees in the portfolio to be 200 with a 60 standard deviation.

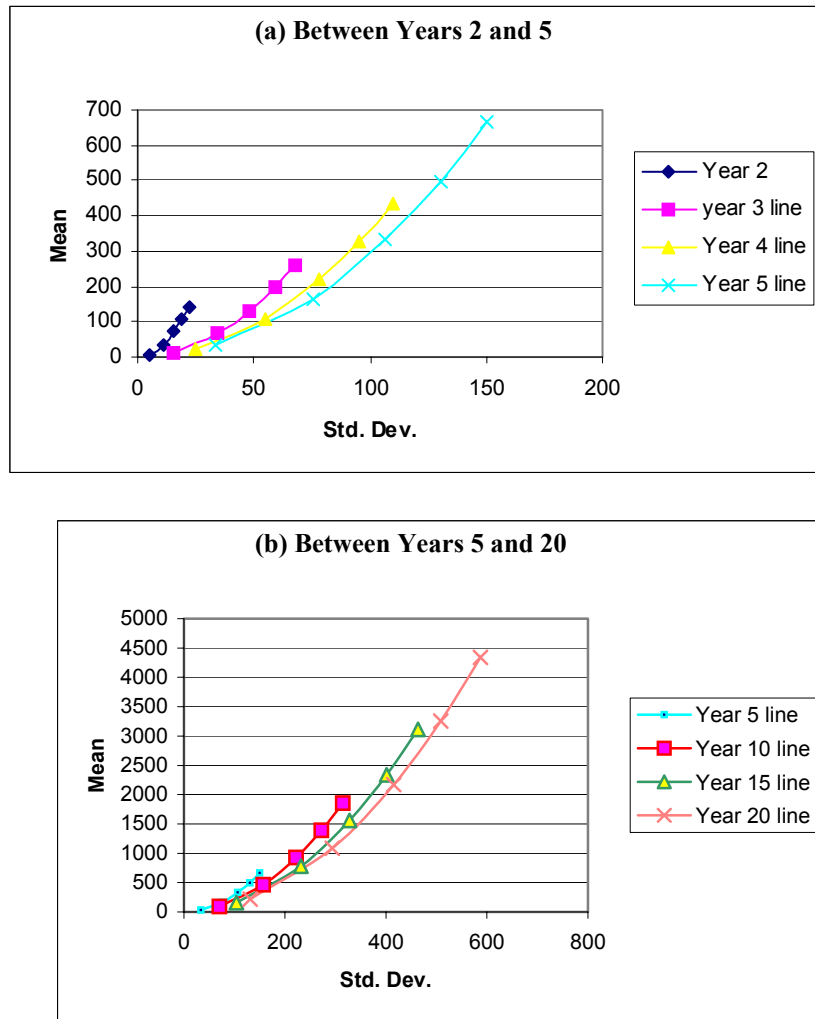


Figure 5: Growth Curves for different Investment Profiles

Even though investments were made for 15 years in a row, we evaluated the *Portfolio* across a 25 year window, because we wanted to see the effects of the

investments in years following the investment itself. The graphs are separated into two so that detail can be observed in early years.

From the graphs, we can see that greater numbers of companies per year reduce risk for a given return in a marginally decreasing manner – as we would expect. This point is evident, for example, if one follows the 10-year curve. A line from the origin to the first point (1 company per year) shows a shallower slope (lower ratio of return to risk) than that for a line from the first point to the second point (5 companies per year). This trend continues with the 3rd, 4th, and 5th investment profiles under consideration.

In Table 1, we provide the same information as in Figure 5. In fact, this table is actually a collection of 3 sub-tables, where each sub-table provides the same values, but at different confidence intervals. Each row in Table 1 indicates a different investment profile. That is, whether 1, 5, 10, 15, or 20 companies are launched per year for 15 years, with no further investment. The next column indicates the corresponding total number of investments which will be made. For example, if 10 companies are launched every year for 15 years, then a total of 150 investments will be made. The third column gives the average number of employees that will exist in the *Portfolio* after it matures and reaches its peak. This is the sustainable number of long term jobs created by the *Portfolio*. The fourth column divides the long-term jobs by the number of investments which were made.

Investment Profile (IP)	Companies Launched (n°W)	Confidence Interval: 50% (z=0)		Confidence Interval: 84% (z=1)		Confidence Interval: 97% (z=2)	
		Long-Term Jobs $\tilde{X}_P(z=0)$	Jobs / Invest (J ₁)	Long-Term Jobs $\tilde{X}_P(z=1)$	Jobs/ Invest (J ₁)	Long-Term Jobs $\tilde{X}_P(z=2)$	Jobs/ Invest (J ₁)
1	15	184	12.27	52	3.47	0	0.00
5	75	920	12.27	547	7.29	315	4.20
10	150	1839	12.26	1230	8.20	902	6.01
15	225	2759	12.26	1935	8.60	1534	6.82
20	300	3678	12.26	2652	8.84	2188	7.29

Table 1: Simulation Data Table

The first sub-table (third and fourth columns) shows values for a 50% confidence interval - this means that with a probability of $\frac{1}{2}$, there will be at least this many employees in the *Portfolio*. The next sub-table (fifth and sixth columns) provides the same statistical information, but with an 84.1% confidence interval. That is, with probability of 84.1%, there will be at least this many employees in the *Portfolio*. Finally, the last sub-table provides the most stringent confidence interval of 97.4%.

6.2. ROI for Investors (ROII) and the Economy (ROIE)

In this section, we calculate ROII and ROIE based on Table 1 and estimates for the conversion factors F_{ROII} , F_{ROIE} , F_{shape} and M .

In Table 2, we provide likely values for F_{ROII} based on reasonable values for I and V_E . For technology companies especially, acquisition costs are often correlated with company size.⁸ Over the recent decades, most technology company acquisitions have been valued in a range from \$0.75M to \$1.5M per employee. Therefore, we have chosen V_E to range between \$0.75M and \$1.5M. The reader also will observe we chose values for I between \$1.25M to \$5M per investment based on Venture capital norms.

Investment (\$M)	Company value/employee (\$M)		
	0.75	1.00	1.50
5	0.15	0.20	0.30
2.5	0.30	0.40	0.60
1.75	0.43	0.57	0.86
1.25	0.60	0.80	1.20

Table 2: Multiplier F_{ROII} Values

For a baseline analysis, we will use \$1.75M to be the expected investment per company (I) and \$1.0 M is the expected company value per employee (V_E). This leads to an F_{ROII} of 0.57 in Table 2. Other reasonable values of F_{ROII} range from 0.15 to 1.20 and can be used for sensitivity analysis. Naturally, the correct F_{ROII} value will depend on

⁸ Stable and growing companies have revenue stream and industry correlated margins which allow for long term funding of their human capital; which is generally a large portion of their operating budgets.

technology and industry type. Finally, we note that I is an average value.⁹ There is also an implicit assumption that the investors will hold a significant portion of the equity.

For the return on investment to the economy (ROIE), we will need to determine values for F_{ROIE} and F_{shape} . Table 3 provides likely values for F_{ROIE} based on reasonable values for s and I . We use the same values for I as in Table 2. Regarding average yearly salary s , we chose \$50K, \$70K, and \$90K as typical reasonable values. For a baseline analysis, we will use \$1.75M to be the expected investment per company (I) and \$0.07M as the expected salary per employee (s). This leads to an F_{ROIE} of 0.04 in Table 3. Other reasonable values range from 0.01 to 0.072.

Investment (\$M)	Average employee yearly salary \$M		
	0.05	0.07	0.09
5	0.0100	0.0140	0.0180
2.5	0.0200	0.0280	0.0360
1.75	0.0286	0.0400	0.0514
1.25	0.0400	0.0560	0.0720

Table 3: F_{ROIE} Values

From the case data, the authors noted that the number of employees is linearly increasing to peak values in year 20 with all of these investment profiles. After year 20, we noted that size is maintained without significant increase or decrease. We can calculate F_{shape} in accordance with Equation (17) by dividing the area under the Portfolio growth curve by the peak number of employees. The area is ($\frac{1}{2} * 20 * \text{peak number of employees}$) for the first 20 years. For the remaining years, we discount the remaining area to be 10 times the 1 year peak because of the long term risk. Therefore we assume F_{shape} to be 20. We must also account for a multiplier effect which describes that these funds continue to circulate in the local economy which we have estimated to be $M = 1.56$.¹⁰

Table 4 provides baseline ROII and ROIE given the investment profiles of the case study and the baseline assumptions above. For example, if we inspect the portion of

⁹ For example, it allows for multiple companies (say 4) to be funded at post seed levels of .5M, with 1 going on to receive the remainder of $4 \times 1.75M$, which is \$6M.

¹⁰ This is a standard Money Circulation multiplier in economics theory and is parameterized by the assumption that 36 cents on every dollar spent in the local economy are re-circulated. That is $M = 1/(1-0.36) = 1.56$.

the table marked for a confidence interval of 84%, we see that investors would see a 467% return on their investment with 10 companies created per year.¹¹ The community would see an ROIE of 1023%.¹² As indicated by the confidence interval, returns would be equal or better with a probability of 84%.

Confidence Interval: 50%

Investment Profile (IP)	Companies Launched	Long Term Jobs	Jobs per Investment	Total Investment ¹³ (\$M)	Baseline ROII	Baseline ROIE
n ^o	n ^o W	$\tilde{X}_P(z=0)$	J _I	\$1.75(n ^o W)		
1	15	184	12.27	26.25	699%	1531%
5	75	920	12.27	131.25	699%	1531%
10	150	1839	12.26	262.50	699%	1530%
15	225	2759	12.26	393.75	699%	1530%
20	300	3678	12.26	525.00	699%	1530%

Confidence Interval: 84.1%

IP	Companies Launched	Long Term Jobs	Jobs per Investment			
1	15	52	3.47	26.25	198%	433%
5	75	547	7.29	131.25	416%	910%
10	150	1230	8.2	262.50	467%	1023%
15	225	1935	8.6	393.75	490%	1073%
20	300	2652	8.84	525.00	504%	1103%

Confidence Interval: 97.7%

IP	Companies Launched	Long Term Jobs	Jobs per Investment			
1	15	0	0	26.25	0%	0%
5	75	315	4.2	131.25	239%	524%
10	150	902	6.01	262.50	343%	750%
15	225	1534	6.82	393.75	389%	851%
20	300	2188	7.29	525.00	416%	910%

Table 4: Total Investment, ROII and ROIE Tables for Various Confidence Intervals

6.3. Case Analysis Using the Approximation Methods:

Using the approximations and results of Section 5, we determine long term behavior and results of the same case study. We have parameterized the Mean Impulse Response

¹¹ Note that the ROII of 467% is equal to the factor F_{ROII} (.57) multiplied by jobs per investment which is 8.2. Other ROII values are calculated in a similar manner.

¹² Similarly, 1023% is calculated by multiplying the F_{ROIE} (0.04) by M (1.56) by F_{shape} (20) and jobs per investment of 8.2.

¹³ We assume \$1.75M as an average investment per company.

(MIR) function and the Deviation Impulse Response (DIR) function to match piecewise linear approximation as per Appendix A. Using these approximations, we show that long term characteristics of the Portfolio are relatively similar to results obtained using the full form developed in Sections 3 and 4.

Investment Profile:

$W = 15$, $n^o = 1, 5, 10, 15$, or 20 for possible investment profiles.

MIR and DIR parameters (From Figures 2A and 3A in Appendix A):

$\mu_{\max} = 12.0$, $t_{\mu}^o = 4.5$ years, $\sigma_{\max} = 35$, $t_{\sigma}^o = 7$ years.

Table 5, below, is populated with previous results compared with results from the linear piece-wise approximation.

<u>Confidence Interval: 50%</u>			Exact			Approximated		
Investment Profile (IP)	Companies Launched	Jobs per Investment	Baseline ROII	Baseline ROIE	Jobs per Investment	Baseline ROII	Baseline ROIE	
n^o	n^oW	J_1			J_1			
1	15	12.27	699%	1531%	12.00	684%	1498%	
5	75	12.27	699%	1531%	12.00	684%	1498%	
10	150	12.26	699%	1530%	12.00	684%	1498%	
15	225	12.26	699%	1530%	12.00	684%	1498%	
20	300	12.26	699%	1530%	12.00	684%	1498%	
<u>Confidence Interval: 84.1%</u>								
IP	Companies Launched	Jobs per Investment						
1	15	3.47	198%	433%	2.96	169%	370%	
5	75	7.29	416%	910%	7.96	454%	993%	
10	150	8.2	467%	1023%	9.14	521%	1141%	
15	225	8.6	490%	1073%	9.67	551%	1206%	
20	300	8.84	504%	1103%	9.98	569%	1245%	
<u>Confidence Interval: 97.7%</u>								
IP	Companies Launched	Jobs per Investment						
1	15	0	0%	0%	0.00	0%	0%	
5	75	4.2	239%	524%	3.92	223%	489%	
10	150	6.01	343%	750%	6.28	358%	784%	
15	225	6.82	389%	851%	7.33	418%	915%	
20	300	7.29	416%	910%	7.96	454%	993%	

Table 5: Summary of Approximation Results

We note that these results are valid for $t > W + \max(t_{\mu}^o, t_{\sigma}^o)$ which is 22 years after the first investment in this case. However, of significant interest is that this quick approximation matches the results in Table 4 reasonably well.

We conclude this section with a sensitivity analysis of ROII and ROIE against environmental variables. In Figure 6, we observed the baseline ROII (with a confidence interval $z=1$) increases with diminishing returns as the number of investments ($n^o W$) increases. The same trend is also observed with a range of environmental variables denoted by F_{ROII} in accordance with Table 2. We note that various combinations of company value per employee and investment per company can significantly alter the expected ROII. Similar sensitivity analysis has also been provided in Figure 7 by observing ROIE and the corresponding effects from varying F_{ROIE} .

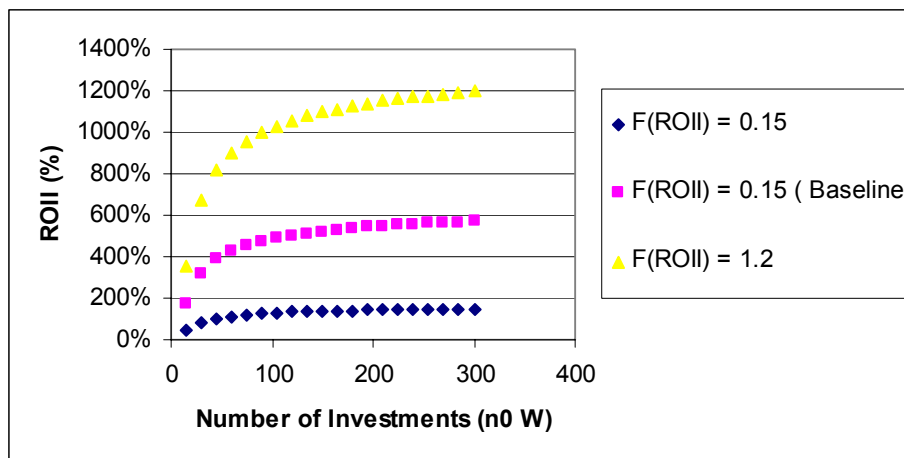


Figure 6: Sensitivity of ROII from F_{ROII}

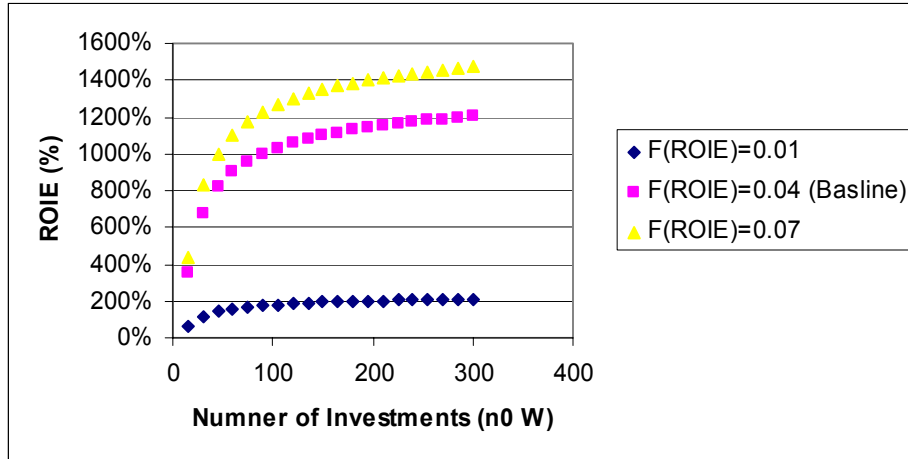


Figure 7: Sensitivity of ROIE from F_{ROIE}

7. Summary and Conclusion

In this paper, we have outlined a general method for predicting growth in a *Portfolio* of companies in a geographic region. The method uses characteristic functions of mean and deviation response to predict sample statistical properties of a firm's growth. Driven by an investment profile, we are able to calculate the statistical properties of the entire *Portfolio* including effects from diversification across multiple, independent, and similar types of investments. We have also illustrated the proposed approach using a realistic case example. In this example, we see direct results of 5 possible investment profiles covering a period of 15 years of investment. A fundamental result of this paper is that the growth of a region or industry can be analyzed with a novel, single closed form economic model.

In both the model as well as the case study, we have noted a number of important effects: First, we have observed reductions in risk from diversification across greater numbers of investments. From an economic development point of view, this helps planners understand how many investments must be made each year in order to benefit from diversification gains. We have also observed large returns on behalf of the community as opposed to only the investors. In the model, we have also separated environmental variables, growth characteristics, and investment profile variables. In this manner, we are able to demonstrate sensitivities and effects across each of these categories. Variation of firm growth characteristics may be represented by MIR and DIR

functions alone. The investment profile formulation allows for variation in total number of investments and time period of those investments. Finally, we see that variations in environmental variables such as Investment required per company (I), average salary (s), and company value per employee (V_E) have significant effects on ROI for investors (ROI) and for the economy (ROIE). In short, the integration of these three distinct driving forces into one single model provides a fundamental analytical tool for evaluating the economic development potential due to new venture creation and entrepreneurial activities in general.

Finally, prior work in economic development suggests that externalities have a strong influence on economic regional growth (Kangasharju, 2001); however, subsequent studies indicate that such externalities are not important and are not sources of permanent growth (Glaeser et al., 2001).¹⁴ In this paper, we have not considered the influence of external factors while creating the growth model of a region. A closer look at the parameters that encourage investment may be one practical extension to this research. Another related area which might deserve some attention is the cost and benefit analysis associated with community-based support such as new business incubators.

¹⁴ Externalities such as a powerful politician bringing a “non-predicted” industry to the region (e.g. Senator Byrd in West Virginia) or the demise (natural or political) of such a powerful figure may upset the forecasted path of growth, as predicted in our proposed model.

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Appendix A: Parameterization of the Impulse Response Functions – MIR & DIR

In this appendix, we consider an empirical method of finding impulse response functions. We begin by building an empirical simulation table which is illustrative of company growth from which the impulse response functions can be estimated. The tables must match our expectations with the known rules and referenced statistics, which have been gathered via literature search. Figure 1A is constructed to characterize “high growth” companies and is based on referenceable rules of thumb and assumptions. These rules and assumptions will translate into “anchor points” for the simulation table. The table is arranged into five columns representing the number of employees in the company partitioned in the following arbitrary sets: failed companies, 1 to 10 employees, 10 and 50 employees, 50 to 100 employees, and 100 and 500 employees. In each of these columns we seek to estimate a “probability” or “fraction of companies” which would grow to the size indicated by the column (i.e. fraction of companies between 1-10, 10-50, 50-100, and 100-500 employees respectively). The very first column represents the proportion of companies that have failed and since ceased to exist. The sum of all probabilities of elements along a row must be equal to 1. We assume the likelihood of companies over 500 employees to be negligible in this study. The simulated table shows the growth of the start-up from year 1 to 25. The rows of the simulation table are indexed by t , the number of years since the inception of the company.¹⁵

The following assumptions and rules of thumb listed below will be used as anchor points for the “High Growth Empirical Simulation Table” (Nesheim, 1992; Case, 1995):

- **Year 1**
 - We assume that a high growth potential venture becomes operational in year one and that the initial size will always be between 1-10 employees with a mean of 4 employees. Represented by the anchor point 1.
- **Year 2**
 - We assume 50% of the companies started fail before the second year of operations (Case, 1995). This is mapped in the second row of the simulation table and is represented by anchor point 2.
- **Year 3**

¹⁵ Note that this is one method of characterization and there may be other different forms of characterizations as well.

- We assume that is possible for a small fraction of high growth companies grow beyond 50-employee mark only by year 3. Represented by 0s in years 1 and for the 50-100 column (this is based on the author's experiences only).
- **Year 5**
 - By the end of the 5th year, 60% of the promising companies fail, while about 10% of the companies have had or will soon have an IPO or M&A (Nesheim, 1997). In other words, these are successes from a venture capital perspective. The row corresponding to year 5 shows 10% success rates and 60% failure rates accordingly and is shown by anchor points 3 and 5.
 - Of the remaining 30% in year 5, we assume a 2 to 1 ratio of companies sized 1-10 vs. 10-50 (anchor points labeled by 4).
 - We have linearly interpolated backwards from the anchor points in year 5 to previously listed anchor points in years 3, 2, and 1 in all the columns.
- **Year 15.**
 - Based on collected regional data as well as the author's experiences, we assume that the original 10% of the highly successful companies in year 5 continue to operate with approximately 1 in 10 continuing to grow past the 100 employee size. This is reflected in anchor points 6 and 7 and linear interpolation is used between years 5 and 15 for these columns.
- **Year 25**
 - By year 25, about 75% of all companies started in year 1 have failed. (Case, 1995). Assume failure rates to be linearly interpolated between years 5 and 25. Shown by anchor point 8.
 - We assume in year 25, the steady state distribution to be 10%, 7%, 7%, and 1% for sizes 1-10, 10-50, 50-100, and 100-500 respectively as represented by anchor points. This represents stabilization of the largest companies and maturation and a possible small decline in midsize businesses.
- All other points are linearly interpreted.

Years	Failed	1 to 10	10 to 50	50 to 100	100 to 500	Mean	Variance	Std. Dev.
1	0	1	0	0	0	5.00	0.00	0.00
2	0.5	0.475	0.025	0	0	3.13	24.61	4.96
3	0.5333	0.3834	0.05	0.0333	0	5.91	206.92	14.38
4	0.5666	0.2918	0.075	0.0666	0	8.70	373.66	19.33
5	0.6	0.2	0.1	0.1	0	11.50	525.25	22.92
6	0.61	0.1915	0.0985	0.099	0.001	11.64	604.88	24.59
7	0.62	0.183	0.097	0.098	0.002	11.78	684.47	26.16
8	0.63	0.1745	0.0955	0.097	0.003	11.91	764.03	27.64
9	0.64	0.166	0.094	0.096	0.004	12.05	843.55	29.04
10	0.65	0.1575	0.0925	0.095	0.005	12.19	923.03	30.38
11	0.66	0.149	0.091	0.094	0.006	12.33	1002.47	31.66
12	0.67	0.1405	0.0895	0.093	0.007	12.46	1081.87	32.89
13	0.68	0.132	0.088	0.092	0.008	12.60	1161.24	34.08
14	0.69	0.1235	0.0865	0.091	0.009	12.74	1240.57	35.22
15	0.7	0.115	0.085	0.09	0.01	12.88	1319.86	36.33
16	0.705	0.1135	0.0835	0.088	0.01	12.67	1312.40	36.23
17	0.71	0.112	0.082	0.086	0.01	12.47	1304.85	36.12
18	0.715	0.1105	0.0805	0.084	0.01	12.27	1297.22	36.02
19	0.72	0.109	0.079	0.082	0.01	12.07	1289.51	35.91
20	0.725	0.1075	0.0775	0.08	0.01	11.86	1281.72	35.80
21	0.73	0.106	0.076	0.078	0.01	11.66	1273.84	35.69
22	0.735	0.1045	0.0745	0.076	0.01	11.46	1265.89	35.58
23	0.74	0.103	0.073	0.074	0.01	11.26	1257.85	35.47
24	0.745	0.1015	0.0715	0.072	0.01	11.05	1249.73	35.35
25	0.75	0.1	0.07	0.07	0.01	10.85	1241.53	35.24

Figure 1A: Simulation Table for High Growth Companies

The weighted mean, variance and standard deviation are estimated from the simulation table; midpoint values are used for each employee range in performing these calculations. The columns for mean and standard deviation are the MIR and DIR estimates.

For promising start-ups the MIR can be estimated from the table, shown in Figure 1, by the function below:

$$\mu_h(t) = [5, 3.13, 5.91, 8.70, 11.50 \dots 11.26, 11.05, 10.85]$$

Similarly, from the table, we estimate the DIR to be as follow:

$$\sigma_h(t) = [0, 4.96, 14.38, 19.33, 22.92 \dots 35.47, 35.35, 35.24]$$

A similar table can be constructed for the marginal growth companies using the same approach. The Mean Response functions for High Growth and Marginal Start-Ups are compared in Figure 2A. As we would expect, high growth companies have higher expected numbers of employees and growth rates. Note that these averages include the companies, which have failed, which is not generally accounted for in surveys of company size. For high growth companies, most of the growth is in the first 5 years, after which growth is relatively flat. For marginal companies, our simulation indicates a relatively flat growth in the first 5-10 years and the possibility of larger company sizes as the company matures.

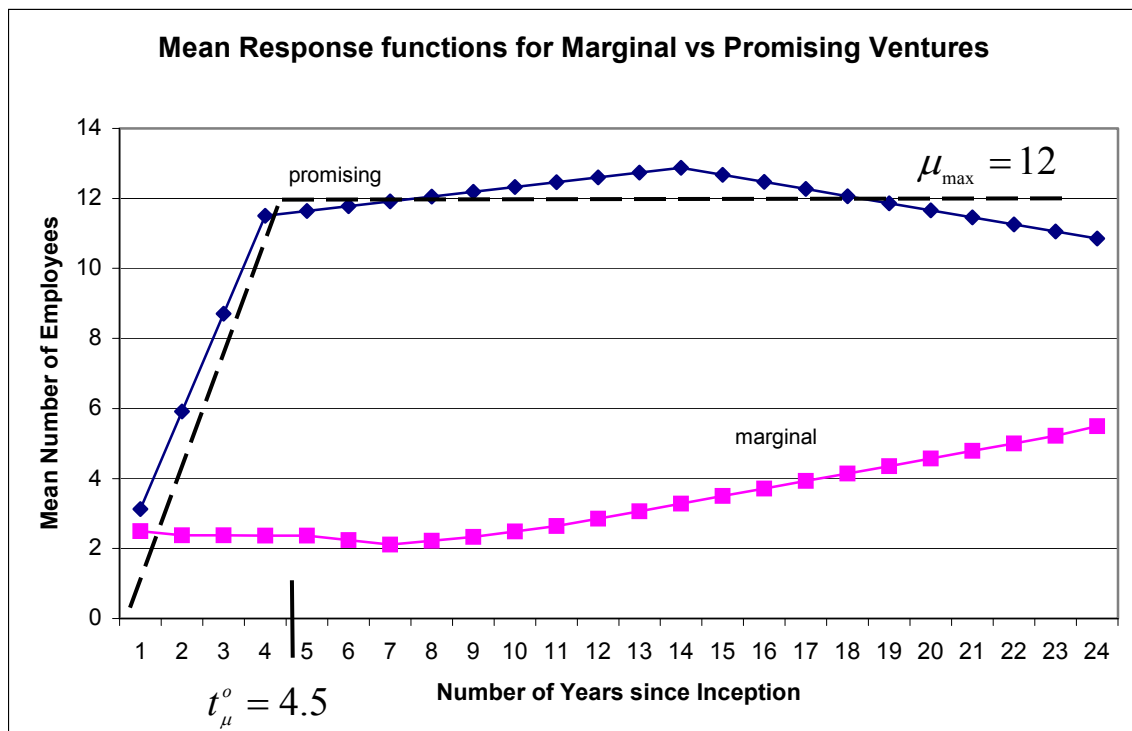


Figure 2A: Mean Impulse Response (MIR) Functions

The Deviation Response functions are compared in Figure 3. Again, as expected, with age, companies disperse in size and their deviations become greater. High growth companies tend to have greater variances than marginal growth companies.

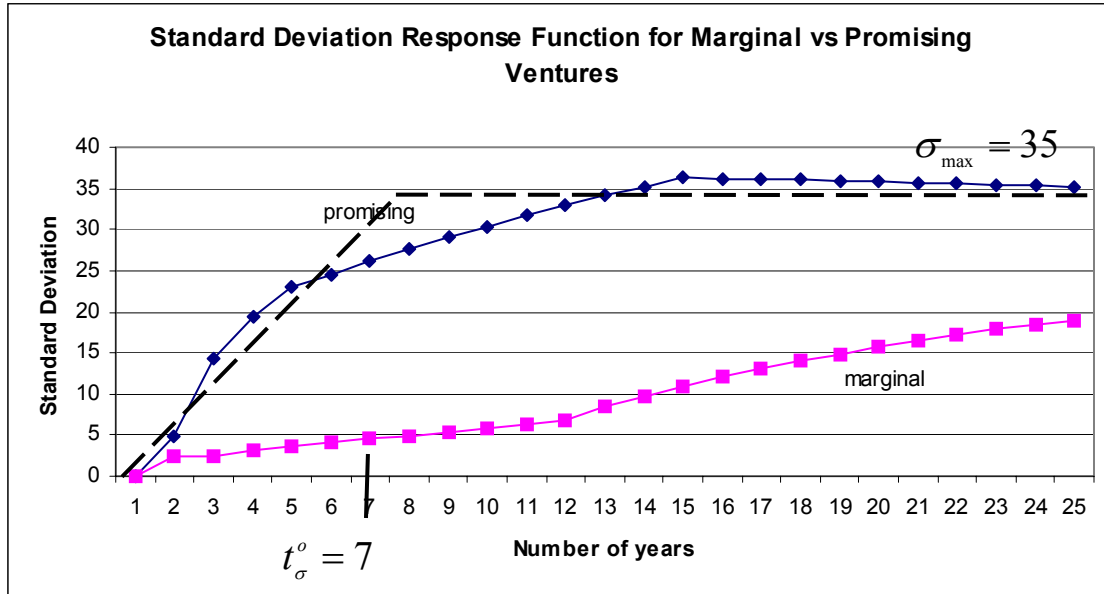


Figure 3A: Standard Deviation Response Functions

In both Figure 2A and 3A, we have overlaid a rough, piece-wise linear approximation using dotted lines. We illustrate that MIR reaches a peak of approximately 12 after 4.5 years. Additionally, the approximated DIR reaches a peak standard deviation of 35 after approximately 7 years. We have used these rough, linear approximations for MIR and DIR in Section 5 of the paper to make the general model more tractable. In doing so, we also found that the results based on these approximations were reasonably close to the exact values based on results of section 3 and 4.