Simple Models of Hierarchical Organizations

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Abstract

This paper develops analytical models for analyzing hierarchical organizations, through the creation and use of little models of organizational interaction to balance the communication time within and between organizational hierarchies. Then it examines these theoretical models on the basis of a data set of 678 school systems in Texas. Analysis of the dataset suggests that, in general, simple models of hierarchy can help understand or compress empirical results. In particular, the communication time ratio (between and within hierarchies) and communication topology ratio are important factors in determining span of control. Furthermore, the school dataset shows that employees, on average, spend more time communicating with superiors compared to the time they spend communicating to other team members in the same hierarchy, while the communication topology is almost the same. The paper concludes by proposing a hypothetical managerial framework for characterizing different types of organizations (or industries) based on these ratios. This framework may help management better understand the appropriate communication time and topology depending on the type of organization.

Keywords: Span of Control, Hierarchy, Organizational Interaction.

1. Introduction

Organizational theorizing has become either overly complex, computational and mathematical (e.g., Qian, 1994; Carley and Prietula, 1994; Huberman and Hogg, 1995; Jin and Levitt, 1996), or it is entirely qualitative (e.g., Woodward, 1980; Meier & Bohte, 2000). Straightforward attempts to build simple quantitative models such as those proposed by Graicunas in the early 1900s were prematurely abandoned (Graicunas, 1937). However, there is a growing movement back toward simple, quantitative organizational modeling (Meier and Bohte 2003). In this paper we will re-visit the span of control (SOC) principle by developing simple models of hierarchical organizations to determine optimal SOC. Then, we examine and validate these models empirically using an existing SOC dataset of 678 Texas school districts.

The span of control (SOC) is a simple managerial construct which identifies or regulates the amount of direct supervision (and hence interaction or contact) that exists between a superior (i.e. a manger or supervisor) and his direct subordinates within an organization. The SOC principle postulates that limiting the number of subordinates (i.e. reporting to a single manager or supervisor) between four and six improves organizational performance (Koontz and O'Donnell, 1959; Bell, 1967, Williamson, 1975; Perrow, 1986).

Although no agreement has been reached about limiting the span of control and what is the optimal span, there is a general agreement among most organizational theorists that the optimal span of control depends on the nature of the organization, nature/scope/scale of work/tasks processed, abilities and style of the superior, and the amount of interaction / coordination that the nature of work in the organization requires (Bell, 1967; Mackenzie, 1974; Hattrup and Kleiner, 1993; Klein, 2001; Nickols, 2003). Few simple qualitative models were proposed in the first half of the twentieth century (Hamilton, 1921; Graicunas, 1937; Gulick, 1937; Urwick, 1956), but these approaches were prematurely abandoned due to a strong critique by Herbert Simon, who called into question many principles of scientific management (Simon, 1946). More recently, numerous detailed mathematical models and empirical investigations have been proposed and we will review many of these in Section 3. In this paper, we try to take an intermediate approach by resurrecting an analytical way of thinking about organizations through the creation and use of little models of organizational interaction (i.e., balancing the interaction / communication time/effort within and between organizational groups) in order to calculate the optimal SOC.

The tendency among engineers to build and use simple models first is not completely a new idea. The field of structural mechanics is a case in point. The simple model of cantilever beam bending, $\delta = PL^3/3EI$, is a simple gross model of beam response to load.[1] Although the model is simple, it is reasonably accurate and, more importantly, gives a great deal of insight into the effect of different variables on beam response. Moreover, the simple model can be used to reason qualitatively about structural response of structures that do not identically fit the underlying assumptions of the model (constant cross-section, modulus, small deflection, etc.). Of course, structural engineers now also have access to detailed computer methods such as finite element methods (FEM) to give detailed predictions about structural performance, but engineers are still taught the little models first, and when reasoning about a structural problem they remain an invaluable tool even with FEM codes sitting on every structural engineer's desktop computer.

Just as progress in mechanics required starting from the simplest of beginnings, our view is that quantitative approaches to organizational understanding must be stripped of all unnecessary complexity. In other words, previous efforts failed (or were disappointing) not because they were simple, but not simple enough. The immediate goal of this paper is to show that a simple model of an organization can provide many useful insights regarding an optimal SOC based on few parameters like the size of the organization and communication time between superiors and subordinates. These models are easier to construct and analyze compared to other more complex (and possibly more accurate) models of the organization which requires a myriad of input parameters and high computation cost. Longer term, we believe that a collection of such little models can be used for quantitative understanding and qualitative organizational insight, in much the same way that a structural engineer routinely turns to the simple models of the mechanics of materials.

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[1] Where δ is beam end deflection, P is the point load at the beam end, L is the beam length, E is the modulus elasticity of the beam material, and I is the moment of inertia of the beam cross-section.

Our viewpoint is that more of this sort of thing is necessary, and here we argue for stripping organizational analytics down to the barest possible bones. Of course, just as strength of materials style models coexist with the most sophisticated FEM codes, we expect the approach here to coexist with and complement more intricately detailed simulation and optimization approaches to organizational understanding. The benefits of quantitative and qualitative understanding of simplified models are indispensable in the physical domain, and we believe they will be just as valuable in the organizational domain once enough of them are developed and methods are available for their integration and interpretation.

The rest of the paper proceeds as follows. In Section 2, we review a subset of organizational theory literature that is directly related to the study of spans of control. In section 3, we propose a new model of organizational design and SOC based on rigorous (but simple) mathematical constructs. Sections 4 and 5 empirically investigate the validity of our proposed model using a sample dataset collected by Meier and Bohte (2000, 2003) on 678 Texas school districts. We conclude the paper in Section 6 by proposing a hypothetical organizational framework and note some limitations and extension of the current work.

2. Literature Review

There is a large body of literature about public administration and organizational design, in general. However, our review here focuses on a subset of this literature that bears most direct relationship to the study and analysis of spans of control. This literature can further be divided into two streams: (a) empirical research directed towards the investigation of actual spans in real organizational and the determinants of spans, and (b) formal mathematical models investigating optimal organizational structures and optimal spans. A third subsection will be devoted to describe our earlier experience with building simple organizational models that constitute a starting point for the model proposed in this paper.

2.1. Empirical Studies

It is frequently cited that the optimal SOC is a number around five, but there has been no conclusive evidence that this is a universal case. For instance, data on the spans of control in colleges and small companies were collected and showed a median of 5-7 (with 95% confidence) in colleges and 4-7 in small companies (Entwisle and Walton, 1961). On the other hand, some researchers reported spans of up to thirty (Udell, 1967).

Many studies have also been conducted to explore the differences in these spans. For instance, Bell (1967) found correlations between the complexity of tasks performed by subordinates and a limited supervisory SOC (based on a study of 33 departments in a general community hospital). The data further indicated that closeness of supervision in unrelated to SOC and the more complex the supervisors' tasks, the fewer are the subordinates they regulate (Bell, 1967). Blau (1968) reported on a study of 250 government agencies confirming that organizations requiring higher qualifications of their personnel are more decentralized, exhibit narrow spans of control, and have large number of managerial levels in their hierarchy. Ouchi and Dowling (1974) surveyed 78 retail department stores and reported that they found typical supervisors managed 8.7 subordinates on average and spent around 50% of their time on supervision.

Finally, Woodward (1980) conducted a study of over 200 British industrial firms and found that variations in spans of control were present across the different types of firms; however, within each category, successful firms used similar spans.[2] More recently, Meier and Bohte (2000, 2003) collected data on 678 Texas school districts and found that the optimal SOC is dependent on three factors: diversity of functions performed by subordinates, tenure of subordinates, and the size of the organization.[3]

2.2. Mathematical Models

As we discussed in the previous section, numerous empirical investigations and treatments for the question of limited or optimal SOC have been reported in the literature. However, these studies were generally inconclusive and too imprecise to permit testable implications to be derived. Consequently a burst of detailed mathematical optimization and computational models have been developed to remedy this situation. In this section we will focus our review on simple models of the organization that are somewhat generic and require the least amount of computational effort; however, provide meaningful managerial insights to help guide organizational thinking and design. We will refrain from discussing other more detailed models of the organization that tend to be tailored for specific organizational settings, requiring numerous organizational assumptions and variables for their development and analysis (e.g., Keren-Levhar 1979; Qian, 1994; Huberman and Hogg, 1995; Jin and Levitt, 1996; Huberman and Loch, 1996; Nasrallah and Levitt, 2001).

One of the earliest simple mathematical models is the one introduced by Graicunas (Graicunas, 1937; Urwick, 1956). He argued that a superior should manage not only direct interaction with subordinates, but also interactions resulting from different groupings of subordinates. That is, if the SOC is three, then a superior will manage three direct interactions in addition to seven group interactions.[4] This leads to the fact that the number of relationships/interactions that a superior must manage increases dramatically (and exponentially) after four subordinates. Thus, a span of control around 4 subordinates is about right, according to Graicunas.

Mackenzie (1974) focused on the determinants of and the calculation for the maximum span of control (instead of an optimal one). Mackenzie's model assumed that any person in a hierarchical organization spends time either working on his/her own task or interacting with others. Furthermore, these interactions can be either with supervisors (at

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[2] Woodward (1980) classified the investigated firms into three types: unit production firms (i.e. shipbuilding), mass production firms, and continuous production firms (i.e. chemical).

[3] The more diverse group a supervisor oversees, the smaller the SOC. The more time subordinates have been doing the same job, the less supervision they require and consequently a large SOC is possible. Finally, the larger organizations tend to use specialists instead of generalists which allow wider spans.

[4] We use the term "direct" to mean one-to-one interaction between a superior and a single subordinate. Alternatively, by "group" we mean interactions that exist between a superior and a particular grouping of subordinates. Therefore, the total number of interactions is: $\sum_i C_i^n = 2^n$.

a higher level of the hierarchy), colleagues (at the same level of hierarchy), or subordinates (at a lower level of hierarchy). Thus, considering the time allotted for a person to finish a specific task, an upper bound exists for the amount of interaction allowed without jeopardizing the completion of the assigned task. Along similar lines, Keren and Levhar (1979) computed the optimal SOC for each level in the hierarchy based on the relative wage rates between supervisors and subordinates. They found that the span of control increases as one goes from the top of the hierarchy towards the bottom, and constant spans of control are optimal only when wage differences are ignored.

Beckmann (1988) wrote an elaborate study of organizational analysis, providing many mathematical formulations for determining the optimal size of the organization, optimal spans of control, and quantifying the loss of control in hierarchical organizations. Assuming constant SOC throughout the hierarchy, Beckmann's main conclusion was that the optimal span of control is independent of the size of the organization.

The focus in this section was on models of hierarchical organizations, due to our interest in investigating the SOC construct.[5] However, other non-hierarchical organizational models exist and bear direct relationship to our proposed model, in terms of striking simple tradeoffs. One such model is reviewed next.

2.3. Simple Models of the Organization

In an earlier work, we (Goldberg et al., 2004) investigated the optimal team size in a non-hierarchical organization based on trading off (a) the time required for deciding what to do, and (b) the time required for doing the work. For the base model, we assumed linear increase in decision making time with team size and equal share of work among team members. Thus, for a team of size *n*, the total deciding-doing time becomes:

$$T(n) = T_1 n + \frac{T_2}{n} \qquad (1)$$

Since it is evident from Equation (1) that the deciding term increases with increased team size, and the doing term decreases with an increase in *n*, then a single unique optimum exists as follows:

$$n^* = \sqrt{\frac{T_2}{T_1}} \qquad (2)$$

A generalization for this simple deciding-doing base model can be constructed by using power-law relationships for both the deciding and doing terms of Equation (1). The general power-law deciding-doing model becomes:

$$T(n) = T_1 n^{c_1} + \frac{T_2}{n^{c_2}} \qquad (3)$$

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[5] Classical organization theory assumes that there exists a hierarchy when speaking of a problem related to span of control.

Where c_1 and c_2 are both ≥ 0 and represent various types of relationships between deciding /doing time and team size.[6,7] Again, taking the derivative of Equation (3) and setting it to zero, we recognize that $T(n)$ has a minimum at:

$$n^* = \left(\frac{c_2 T_2}{c_1 T_1} \right)^{\frac{1}{c_1 + c_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Then, the authors formalized the idea of using simple, one-dimensional optimization as a modeling tool in organizational contexts and elsewhere as an elementary optimization problem (EOP). The paper concluded by suggesting how the systematic study of EOPs, other simple models, and patchquilt integration using dimensional analysis may permit the formulation of a more sophisticated quantitative understanding of problems in organizational theory than would otherwise be possible.

3. A Model of Organizational Hierarchy

Following the EOP approach, we propose a simple model of organizational hierarchy. In the deciding-doing model, we assumed the existence of a single team and looked at tradeoffs between decision and execution times in that team. Here we look at the tradeoffs between communication times up and down the organization versus communication across each team.

In this model, we consider an organizational hierarchy, as shown in Figure 1. For such a hierarchy, we find the optimal span of control as a function of the size of the organization, and the relative communication times between and within a hierarchy.

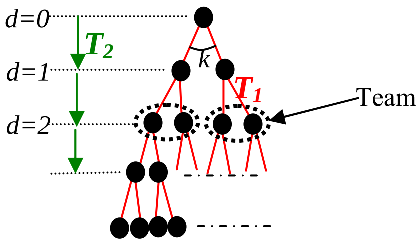

Figure 1: Hierarchical organizational structure

We consider a hierarchical organization of size m (i.e. total members in the organization) and we also assume that this hierarchy has d tiers (counting from the top of the hierarchy

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[6] For instance, $c_1 = 0$ corresponds to constant decision time, irrespective of team size. The case where $c_1 = 1$ corresponds to the base model. The case where $c_1 = 2$ corresponds to a decision process that permitted all pairwise conversations (between team members) to take place.

[7] The case $c_2 = 0$ corresponds to work that cannot be speeded with additional help. The case $c_2 = 1$ is ideal work sharing as assumed in the basic model. The situation with $0 < c_2 < 1$ corresponds to those cases where the addition of workers does not reduce the execution time inversely with the number added. Cases with $c_2 > 1$ correspond to situations where the addition of workers promotes faster times that would occur if the work were split among n workers equally.

to the bottom; at the top of the hierarchy $d = 0$, then 1 and so on). Finally, we assume a constant span of control k exists throughout the hierarchy.

Teamwise, we consider that communication within each team of size k, at all levels of the hierarchy, requires T_1 time units per team member, and that communication takes place simultaneously and in parallel among all teams. Communication between layers of the hierarchy is assumed to require T_2 units per layer. We investigate the efficiency of such organization by considering the various communication links within the hierarchy. Therefore, the total communication time is composed of two components: (a) the time required for hierarchical communication (between teams), and (b) the time required for communication within a hierarchy (within a team), as shown in Equation (5):

$$T = kT_1 + dT_2 \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Let us define S_i as the cumulative number of members (i.e. employees) form the top of the hierarchy ($d = 0$) until (and including) the i^{th} tier (with $S_0 = 1$). Then, the organization size, m, becomes: $m = S_d = 1 + k + k^2 + \ldots. + k^d = \sum_d k^d$. In this hierarchy, it can be easily shown that the following relationships hold:

$$\begin{cases} S_{d+1} = kS_d + 1 \\ S_{d+1} - S_d = k^d \end{cases} \ldots\ldots\ldots\ldots\ldots \quad (6)$$

Equation (6) yields:

$$m = \frac{k^{d+1} - 1}{k - 1} \approx k^d \text{, for large } k. \ldots\ldots\ldots\ldots \quad (7)$$

Finally, solving for d results in:

$$d = \frac{\ln m}{\ln k} \ldots\ldots\ldots\ldots\ldots \quad (8)$$

Substituting Eq'n (8) into Eq'n (5) yields:

$$T = kT_1 + \frac{\ln m}{\ln k} T_2 \ldots\ldots\ldots\ldots\ldots \quad (9)$$

Taking the derivative of Eq'ns (9) with respect to k, setting it to zero, and solving for optimal m yields:

$$m = \exp\left[\frac{T_1}{T_2} k(\ln k)^2\right] = \exp\left[k(\ln k)^2\right]^{T_1/T_2} \ldots\ldots \quad (10)$$

Equation (5) assumes a simple communication topology within teams, which results in a team communication time of kT_1 time units and thus is a linear function of team size, k. However, if we allow a different (e.g., higher order) communication topology to exist among team members, then team communication time becomes quadratic in *k* and the total communication time becomes:[8]

$$T = \left(C_2^k\right)T_1 + dT_2 \approx \frac{k^2}{2}T_1 + dT_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

Substituting Eq'n (8) into Eq'n (11) yields:

$$T = \frac{k^2}{2}T_1 + \frac{\ln m}{\ln k}T_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

Again, taking the derivative of Eq'ns (12) with respect to *k*, setting it to zero, and solving for optimal *m* yields:

$$m = \exp\left[\frac{T_1}{T_2}\left(k \ln k\right)^2\right] = \exp\left[\left(k \ln k\right)^2\right]^{T_1/T_2} \quad \ldots\ldots \quad (13)$$

Both Equations (10) and (13) show that the size of the organization, *m*, grows faster than exponential in the span of control, *k*, and the relative cost of communicating within and between the levels of the hierarchy, T_1/T_2.

Now we consider a generalization of both models (as depicted by Equations 10 and 13) by considering a to be the communication order (i.e. topology) inside the teams and b to be the communication order outside the teams, instead of the linear and quadratic forms used for deriving Equations (10) and (13). Then, the general form for the total communication time (between and within a hierarchy), for large k and d values, becomes:

$$T = \binom{k}{a}T_1 + \binom{d}{b}T_2 \cong \frac{k^a}{a!}T_1 + \frac{d^b}{b!}T_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Taking the derivative of Equation (14) with respect to *k*, setting it to zero, and solving for *m* yields:[9]

$$(\ln m)^b = \frac{\Gamma(a)}{\Gamma(b)} \cdot k^a (\ln k)^{b+1} \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Note that Equations (10) (referred to as **Model 1**) is a special case for the generalized model of Equation (15), where $a = 1$ and $b = 1$. Furthermore, Equation (13) (referred to as **Model 2**) is a special case for the generalized model, where $a = 2$ and $b = 1$.

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[8] We consider a large number of teams and members, which means: $C_2^n \approx \frac{n^2}{2}$.

[9] The intermediate step between Equations (14) and (15) is: $\frac{dT}{dk} \cong \frac{k^{a-1}}{(a-1)!}T_1 - \frac{(\ln m)^b}{(b-1)!} \cdot \frac{1}{(\ln k)^{b+1}} \cdot \frac{1}{k}T_2 \equiv 0$ ·

In the next section, we empirically test the validity and utility of these little models (i.e., Equations 10, 13 and 14) by analyzing an organizational dataset of 678 Texas school districts (Meier and Bohte, 2000, 2003).

4. Empirical Findings

An ideal dataset for validating our proposed models would simply have four variables available: the size of the organization, the span of control, the time consumed for communicating up and down the organizational hierarchy, and the time consumed for communicating within the same hierarchy. In this study, we use the data collected by Meier and Bohte (2000, 2003) from 678 Texas school districts with enrollment of over 500 students. The data cover the years 1994 to 1997 and were pooled to generate a total of 2,712 cases over the timeframe of their study. This dataset is useful for our purposes, but not ideal since it did not include estimates of communication times (within and between hierarchical teams). However, we have utilized two variables from this dataset: the ratio of teachers to school administrators (*k*) and the staff size (*m*). We had to estimate the fraction T_1/T_2 as follows:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\ln m_i}{k_i\ (\ln k_i)^2} \text{ for Model 1, ……………………..(16)}$$

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\ln m_i}{(k_i \ln k_i)^2} \text{ for Model 2, ……………………..(17)}$$

Where m_i and k_i are the given data points, and N is the number of data points.

Once we estimate T_1/T_2, given the organization size m_i, we can then numerically calculate back the *optimal* SOC $\hat{k}_i$ according to our proposed models (Equations 10 and 13). The mean of errors $\frac{1}{N}\sum_{i=1}^{N}|k_i - \hat{k}_i|$ tells us how these models fit the given dataset.

Figure 2 shows a scatter plot of the dataset based on model 1, where the slope of the best-fit line represents the fraction (T_2/T_1). The variation observed around this line is due to differences in the relative communication times. The estimated communication ratio $\frac{T_2}{T_1} = \frac{k(\ln k)^2}{\ln m} = 17.313$, with a mean of errors of 2.0831. This ratio suggests that employees, on average, spend much more time communicating with superiors compared to the time they spend communicating to other team members. We will elaborate further on this observation in the conclusion section.

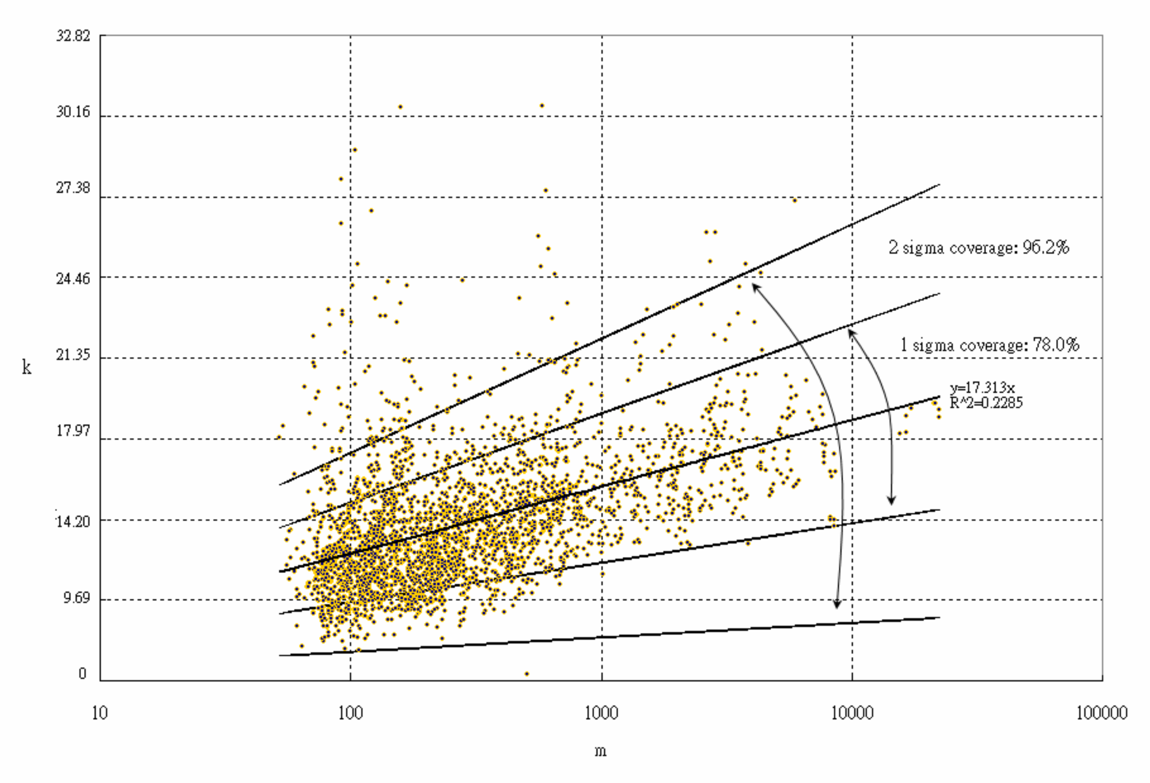

Figure 2: A Scatter Plot of the School Data (Model 1, a=b=1)

(The *x* axis is the organization size (*m*) in logarithmic scale; and the *y* axis is the SOC (*k*) in "$k(\ln k)^2$" scale. The slope of the trend line is T_2/T_1. The plot also shows a good coverage within 2 standard deviations from the trend line)

A similar plot can be obtained for Model 2 with an estimated $T_2/T_1 = 257.1635$, and the mean of errors is 2.2178. Therefore, according to the above mean of errors, model 1 better describes the given dataset, compared to model 2. However, in the next section we investigate how sensitive this conclusion is.

5. Sensitivity to Modeling Parameters

This section investigates whether the general model, with varying exponents *a* and *b* in Equation (15) better describes the given dataset. Assuming that the data came from the same T_1 and T_2 distributions, then the fraction T_1/T_2 can be estimated by:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{(\ln m_i)^b}{\frac{\Gamma(a)}{\Gamma(b)}\cdot k_i^{\,a}(\ln k_i)^{b+1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

If we assume that $b=1$, then the following chart (Figure 3) is drawn by varying parameter a and calculating the means of errors. Figure 3 shows that the minimal mean of errors occurs roughly at $a=1.05$. If we allow 10% error, as the figure shows, both models 1 and 2 are good enough for describing the given data.

When we approximate Equation (15) by $\sqrt[b]{\frac{\Gamma(a)}{\Gamma(b)}\cdot\frac{T_1}{T_2}}\cdot k^{\frac{a}{b}}\cdot(\ln k)=\ln m$, for large b (by assuming $\frac{b+1}{b}\approx 1$). Then, $\frac{T_1}{T_2}$ is estimated by Equation (18), and hence $\sqrt[b]{\frac{\Gamma(a)}{\Gamma(b)}\cdot\frac{T_1}{T_2}}$ is fixed for given m and k. Therefore, the estimated SOC, $\hat{k}_i$, depends only on a/b for a given m_i. Based on the observation, the minimal means of errors should occur roughly at the same a/b values. Figure 4 provides a whole family of models and is drawn by varying b and finding the ratio a/b which yields minimal means of errors. For large b, the minimal errors occur roughly at the same a/b ratio, 0.86 (which is consistent with our original guess is the previous section). Recalling that a is the communication order inside the teams and b is the communication order outside the teams, then the result indicates that the communication order inside the teams is smaller than (but close to) that outside the teams.

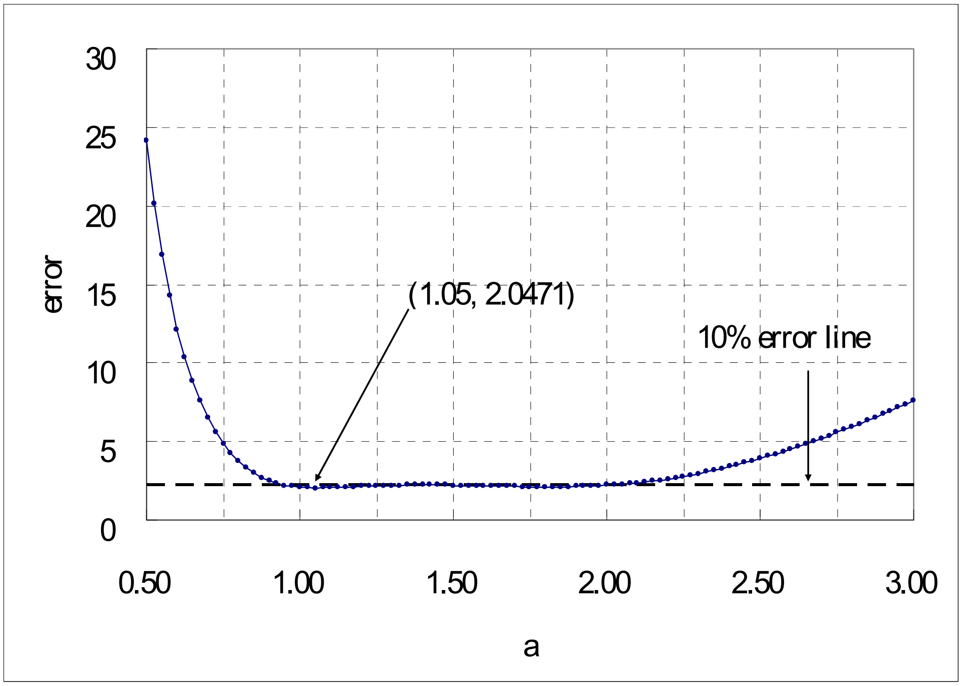

Figure 3: For a fixed $b = 1$, the mean of errors, ($|k-\hat{k}|$) versus varying a. The 10% error line indicates that both model 1 and model 2 are good enough to describe the given data.

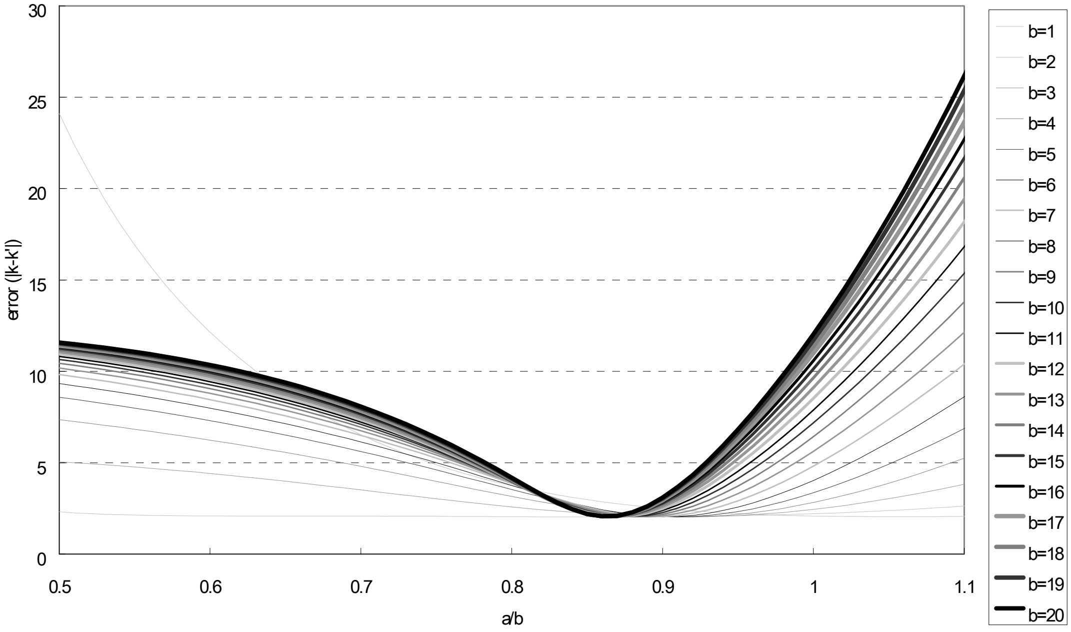

Figure 4: Relation between *a/b* ratios and the errors ($|k-\hat{k}|$) - for different *b* values. The minimal errors take place roughly at the same *a/b* ratio, especially when *b* is large.

6. Insights and Concluding Remarks

This paper introduced a simple model of hierarchical organizations by considering the tradeoff between inter-level and intra-level communication times. These models allowed us to find simple expressions for the optimal span of control. In the simplest setting we assume that communication exists only and directly between a supervisor and her subordinates, without allowing interaction between subordinates (i.e. inter-level communication is a linear function of the SOC, *k*). In the second model, we relax this assumption by allowing communication amongst subordinates, resulting in a quadratic, in *k*, inter-level communication time). These basic models are then generalized by introducing power-law relationships for both the inter- and intra-level communication times.

Then, we empirically investigated the validity and utility of the proposed modeling construct using span of control data from 678 Texas school districts, which included a total of 2,712 cases. Analysis of this dataset using the generalized form of the proposed model showed that the communication order inside the teams is smaller than (but close to) that outside the teams. Furthermore, the communication time ratio (T_2/T_1) was quite large. Though a bit surprising, a closer look into the source of data reveal that this is quite reasonable, especially when considering that teachers are fairly autonomous and consequently school systems have relatively loose coupling between lower level teams, but fairly strong top to bottom communication. In other organizations (and industries), such as design or software development, we could expect this to reverse.

A hypothetical managerial framework could be developed based on our model providing insights and guidance to support organizational design decision-making. Managers could select their appropriate relative communication times (T_2/T_1) and team interaction topology (a) depending on the type of organization. Figure 5 shows such a framework, which makes hypothetical predictions regarding the location of several types or organizations. However, in order to verify these characterizations, we suggest the collection and analysis of a comprehensive dataset across various industries (i.e. organization size, m, span of control, k, communication times, T_1 and T_2, and communication topologies, a and b) to assess the validity of such framework.

Tightly Supervised Relative Communict'n times (T_2/T_1)	Academic / Traditional manufacturing	Military
Relatively Autonomous	Modern manufacturing / Professional services	Design / Innovation
	Simple Team Interaction (a) Complex	

Figure 5: A Managerial Framework for Organizational Design

The above framework assumes low interconnectivity for managerial communication (i.e., $b \approx 1$) which is a reasonable assumption in a pure hierarchy. The relative communication times between and within hierarchies, (T_2/T_1), ranges from relatively autonomous to tightly supervised. The communication topology among teams (a) ranges from simple to complex interaction patterns. To describe the proposed framework, we will start at the top left quadrant and move around it in an anti-clockwise direction.

Our earlier empirical results showed that academic organizations, such as schools, fall in the upper left quadrant with large T_2/T_1 ratio and low team interaction. Traditional manufacturing organizations probably fall in this same category since line workers require minimal interaction with tight hierarchical supervision. Modern manufacturing organizations that promote communication among team members (e.g. to solve problems or suggest improvements) will have a reduced T_2/T_1 ratio and thus may fall in the lower left quadrant. Professional service organizations, such as hospitals and law offices, could also fall in this quadrant, since doctors and lawyers are relatively autonomous and do not interact among themselves much.

The organiztion moves to the lower right quadrant when the requirement for team interaction is increased, such as organizations that are involved in design or innovation.

These organizations are characterized by having fairly interactive teams, with loose hierarchical control. Finally, military organizations seem to fit into the upper right quadrant where a rigid hierarchy exists as orders are pushed down the hierarchy, but team performance is highly dependent on strong coordination among its members.

We intentionally kept our modeling constructs and analysis to a bare minimum in order to extract the most intuitive results that may represent some obvious rules of thumb. There has been progress in understanding organizational theory computationally in recent years, but the busy complexity and one-off nature of those modeling efforts makes us wonder whether there aren't simpler analytical models to help guide our organizational thinking and design. This paper has offered two such models and a generalized form, for developing many more models having various communication intensities within and between the levels of the hierarchy. We believe that a collection of such little models can be used for quantitative understanding and qualitative organizational insight once enough of them are developed and methods are available for their integration and interpretation.

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